Answers:

1. D
2. A
3. C
4. D
5. B
6. C
7. D
8. A
9. A
10. B
11. E
12. C
13. A
14. B
15. C
16. B
17. A
18. B
19. C
20. D
21. B
22. A
23. D
24. B
25. B
26. C
27. A
28. D
29. B
30. C

Solutions:

1. $0 = x^2 - y^2 - 2x + 4y - 8 = (x-1)^2 - (y-2)^2 - 5$, so this hyperbola has center at $(1,2)$.

2. The line with the largest slope would have the highest profit growth rate. In the order given, the slopes are $4/3$, $1/7$, $8/9$, and $6/5$. The largest of these slopes is the first one, which corresponds to Jojo's Coffee.

3. $0 = x^2 + y^2 - 6x - 7 = x^2 + (y-3)^2 - 16$, so the radius of the circle has length 4. The square's side length would equal the diameter, so the side length is 8, making the sought area $8^2 - \pi \cdot 4^2 = 64 - 16\pi$.

4. The equation of the line would be $y - 3 = -\frac{1}{2}(x-2) \Rightarrow 2y - 6 = -x + 2 \Rightarrow x + 2y = 8$.

5. The midpoint is $\left(\frac{6+2}{2}, \frac{m-1+m-3}{2}\right) = (4, m-2)$.

6. The unreduced slope between $(1,2)$ and $(7,6)$ is $\frac{6-2}{7-1} = \frac{4}{6}$, so a point with larger coordinates that has the same unreduced slope through the point $(9,1)$ satisfies $\frac{y-1}{x-9} = \frac{4}{6} \Rightarrow (x,y) = (15,5)$.

7. Distance from the y-axis is governed by the x-coordinate, but this coordinate could be positive or negative. Therefore, the distance is $|x|$.

8. The midpoint between the given foci is the origin, so this is the center. Additionally, the distance between vertices is 4, so $2a = 4 \Rightarrow a = 2$. $c = \sqrt{7}$, so $b^2 = c^2 - a^2 = \sqrt{7}^2 - 2^2 = 7 - 4 = 3$. Since the foci lie on a vertical line, the equation would be $\frac{y^2}{4} - \frac{x^2}{3} = 1$.

9. From the origin to the center, one must move to the right 1 and up 2. Moving another 1 to the right and 2 up, the path finishes at $(2,4)$.

10. Since the two lines intersect $y = x$, the points of intersection satisfy $4x + 2x = 9$

$\Rightarrow 6x = 9 \Rightarrow x = 3/2$ and $2x + x = 6 \Rightarrow 3x = 6 \Rightarrow x = 2$, so P and Q are $\left(3/2, 3/2\right)$ and $(2,2)$, respectively. Using similar triangles with common point $T(0,0)$, the sought ratio is $\frac{3/2}{2} = \frac{3}{4}$.

11. Since $r > 1$, the number under the x term is negative and the number under the y term is positive, so both terms on the left-hand side are negative. Therefore, the left-hand side is not positive while the right-hand side is 1, so there are no points on this graph.

12. $0 = x^2 - 8x + 2y + 7 = (x-4)^2 + 2y - 9 \Rightarrow y = -\frac{1}{2}(x-4)^2 + \frac{9}{2}$, so the parabola's vertex is at the point $\left(4, 9/2\right)$. The parabola opens downward, so because $4p = 2 \Rightarrow p = \frac{1}{2}$, the focus lies $\frac{1}{2}$ unit below the vertex, making $(4,4)$ the focus.

13. This equation can be written as $\frac{(y-1)^2}{16} - \frac{(x+3)^2}{25} = 1$, so the slopes of the asymptotes are $\pm\frac{4}{5}$. Since the asymptotes also pass through the center of the hyperbola, their equations are $y - 1 = \pm\frac{4}{5}(x+3)$.

14. For this equation to represent an ellipse, both $1 - r < 0$ and $r - 3 < 0 \Rightarrow 1 < r < 3$.

15. If these four points are shifted to the left 143 units and down 56 units, the new vertices are at the points $(0,8)$, $(15,0)$, $(0,-8)$, and $(-15,0)$, which clearly yields a rhombus with diagonals of length 16 and 30. Therefore, the area enclosed by the rhombus is $\frac{1}{2} \cdot 16 \cdot 30 = 240$.

16. All points equidistant from the given points are on the line $x = 6$, and both of the given points are a distance of 3 away from this line. To select a point in the fourth quadrant that is a distance of 4 away, move down the line $\sqrt{4^2 - 3^2} = \sqrt{7}$ units. Therefore, the sought point is $\left(6, -\sqrt{7}\right)$.

17. If the third point is (x,y), then $(2,-1)=\left(\frac{3-7+x}{3},\frac{-5+4+y}{3}\right)\Rightarrow(x,y)=(10,-2)$.

18. To get from A to the dividing point, move to the right 2 and down 4. To get from the dividing point to B, move to the right 7 and down 14. Therefore, the ratio of the shorter length to the longer length is $2:7$.

19. Since the equation has horizontal axis of symmetry and vertex at the origin, the form of the equation is $y^2=4px$. Since the parabola passes through $(-2,4)$, we must have $16=4^2=4p(-2)=-8p\Rightarrow p=-2$, so the equation is $y^2=-8x\Rightarrow 8x=-y^2$.

20. $m=\frac{4+3m-(6-m)}{4+m-5m}=\frac{4m-2}{4-4m}\Rightarrow 4m-4m^2=4m-2\Rightarrow m^2=\frac{1}{2}\Rightarrow m=\frac{\sqrt{2}}{2}$ since $m>0$

21. Based on the given information, the ellipse has vertical major axis and center at $(2,-3)$. Additionally, $c=\sqrt{7}$ and $b=3$, so $a^2=b^2+c^2=3^2+\sqrt{7}^2=9+7=16$, meaning the enclosed area is $\pi ab=\pi\cdot 4\cdot 3=12\pi$.

22. $0=2x^2-12x-y+22=2(x-3)^2-y+4\Rightarrow y=2(x-3)^2+4$, which has vertex at $(3,4)$. If every point is moved right 3 units and up 4 units, the new vertex is at $(6,8)$, meaning the equation is $y=2(x-6)^2+8=2x^2-24x+80$
$\Rightarrow 0=2x^2-24x-y+80$

23. $0=ax^2+2y^2-4y+2(1-a)=ax^2+2(y-1)^2-2a\Rightarrow\frac{x^2}{2}+\frac{(y-1)^2}{a}=1$. If $a>2$, then the latus rectum length is $1=\frac{2\cdot 2}{\sqrt{a}}\Rightarrow\sqrt{a}=4\Rightarrow a=16$, which is consistent. If $a<2$, then the latus rectum length is $1=\frac{2a}{\sqrt{2}}\Rightarrow a=\frac{\sqrt{2}}{2}$, which is also consistent.

Therefore, the product of the values is $16\cdot\frac{\sqrt{2}}{2}=8\sqrt{2}$.

24. Subtracting 5 times the first equation from twice the second equation eliminates the x term, yielding the equation $9y=27\Rightarrow y=3\Rightarrow 2x=28\Rightarrow x=14$.

25. The form for conics written in polar form is $r=\dfrac{ep}{1-e\cos\theta}$, where e is the eccentricity. Therefore, $r=\dfrac{1}{2-\cos\theta}=\dfrac{1/2}{1-\frac{1}{2}\cos\theta}$, meaning the eccentricity is $\dfrac{1}{2}$, meaning the conic is an ellipse.

OR

$$2r-r\cos\theta=1\Rightarrow 2\sqrt{x^2+y^2}=x+1\Rightarrow 4\left(x^2+y^2\right)=x^2+2x+1\Rightarrow 3x^2-2x+4y^2=1$$

$\Rightarrow 3\left(x-\dfrac{1}{3}\right)^2+4y^2=\dfrac{4}{3}$, which is the equation of an ellipse.

26. The directrix is $\dfrac{a^2}{c}$ units from the center, running perpendicular to the major axis. $a^2=64$ and $c^2=a^2+b^2=64+225=289\Rightarrow c=17$. Therefore, because the hyperbola opens horizontally, the directrices are vertical lines a distance of $64/17$ from the center. The greater value of k would then be $k=-1+\dfrac{64}{17}=\dfrac{47}{17}$.

27. The equation is $y^2=-4wx\Rightarrow x=-\dfrac{1}{4w}y^2$, so the distance between the vertex and the focus, which is also the distance between the vertex and the directix, is $|w|$. Therefore, the distance between the focus and the directrix is $2|w|$.

28. $0=2x^2+y^2-4x-2y-3=2(x-1)^2+(y-1)^2-6\Rightarrow\dfrac{(x-1)^2}{3}+\dfrac{(y-1)^2}{6}=1$, so the first conic section has center $(1,1)$, $a=\sqrt{6}$, and eccentricity $e=\dfrac{c}{a}=\dfrac{\sqrt{6-3}}{\sqrt{6}}=\dfrac{1}{\sqrt{2}}$. This means the second conic section has center $(1,1)$, $a=\sqrt{6}$, and eccentricity $e=\dfrac{c}{a}=\sqrt{2}$, meaning the second conic section is a hyperbola with vertical major axis. Additionally, for the second conic section, $c=a\sqrt{2}=\sqrt{6}\cdot\sqrt{2}=2\sqrt{3}$, so $b^2=c^2-a^2$ $=12-6=6$, so the equation of the hyperbola is $\dfrac{(y-1)^2}{6}-\dfrac{(x-1)^2}{6}=1$

$\Rightarrow y^2-2y+1-x^2+2x-1=6\Rightarrow x^2-y^2-2x+2y+6=0$.

29. The circumcenter is equidistant from all three vertices, and the points equidistant from the first two points are all on the line $y=2$, so let $(x,2)$ be the circumcenter. Then $(x+3)^2+(2+4)^2=(x-5)^2+(2-2)^2 \Rightarrow 6x+45=-10x+25 \Rightarrow 16x=-20$ $\Rightarrow x=-5/4$, so the circumcenter is at the point $\left(-5/4,2\right)$.

30. The conic section is a parabola, so the eccentricity is 1, regardless of the other details.