Answers:

1. B
2. D
3. A
4. D
5. A
6. B
7. C
8. B
9. D
10. E
11. A
12. B
13. D
14. B
15. B
16. B
17. C
18. A
19. A
20. D
21. E
22. C
23. C
24. E
25. A
26. A
27. C
28. D
29. C
30. A

Solutions:

1. To get from D to A, you must go to the right 12 and down 100. To go three-fourths of the way, you would go to the right 9 and down 75, so the new point is $(-2+9, 22-75)=(7,-53)$.

2. The slopes of the two lines are $-\tfrac{3}{2}$ and 1, so the acute angle θ satisfies
$$\tan\theta=\frac{-\tfrac{3}{2}-1}{1+\left(-\tfrac{3}{2}\right)(1)}=\frac{-\tfrac{5}{2}}{-\tfrac{1}{2}}=5.$$

3. $\dfrac{y-2}{x-1}=\dfrac{1}{2}\cdot\dfrac{y-5}{x-2}\Rightarrow 2xy-4y-4x+8=xy-y-5x+5\Rightarrow xy+x-3y+3=0$, and the slopes would be undefined if x were equal to 1 or 2, so $x\neq 1$ and $x\neq 2$.

4. The tangent line to a polynomial equation at the origin is always the homogeneous linear portion of the polynomial equation, so it is $2x-3y=0\Rightarrow y=\dfrac{2}{3}x$.

5. The slope of the line would be $\tan 60^\circ=\sqrt{3}$, so the equation of the line is $y-6=\sqrt{3}(x-4)$.

6. $\overrightarrow{AB}=\langle -10-6, 43-15\rangle=\langle -16, 28\rangle$

7. The midpoint of the segment is $(5,2)$ and the slope of the segment is $\dfrac{-2-6}{6-4}=-4$, making the slope of the perpendicular bisector $\tfrac{1}{4}$. Therefore, the equation of the perpendicular bisector is $y-2=\tfrac{1}{4}(x-5)$, which makes the y-intercept satisfy $y=\dfrac{1}{4}(0-5)+2=\dfrac{3}{4}$.

8. The conic is given by $\dfrac{x^2}{\tfrac{2}{3}}-\dfrac{y^2}{\tfrac{2}{5}}=1$, so $c=\sqrt{\dfrac{2}{3}+\dfrac{2}{5}}=\sqrt{\dfrac{16}{15}}=\dfrac{4\sqrt{15}}{15}$, and because $a=\sqrt{\dfrac{2}{3}}=\dfrac{\sqrt{6}}{3}$, $e=\dfrac{c}{a}=\dfrac{\tfrac{4\sqrt{15}}{15}}{\tfrac{\sqrt{6}}{3}}=\dfrac{12\sqrt{5}}{15\sqrt{2}}=\dfrac{2\sqrt{10}}{5}$.

9. The De Longchamp's point is the reflection of the orthocenter across the circumcenter, so the circumcenter's distance to the orthocenter is the same as the De Longchamp's point's distance to the circumcenter, which is given as 9.

10. $\left(\frac{0+20-8}{3},\frac{10+2-20}{3}\right)=\left(4,-\tfrac{8}{3}\right)$

11. Using the Shoelace method, the area enclosed by this quadrilateral is

$$\begin{array}{r|rr|r} & 0 & 10 & \\ 200 & 20 & 2 & 0 \\ -16 & -8 & -20 & -400 \\ -20 & 1 & -1 & 8 \\ 0 & 0 & 10 & 10 \\ 164 & & & -382 \end{array} \Rightarrow A=\frac{1}{2}\left|164-(-382)\right|=\frac{1}{2}\cdot 546=273.$$

12. $B^2-4AC=(-2)^2-4(1)(1)=0$, so because the conic is non-degenerate, it is a parabola.

13. This is the definition of a hyperbola.

14. The vertices of this hyperbola occur when $y=-x$, so the vertices are at the points $\left(\mp 6\sqrt{2},\pm 6\sqrt{2}\right)$, making $a=12$. Since the axes are the asymptotes of this graph, this makes $b=12$, which makes $c=\sqrt{12^2+12^2}=12\sqrt{2}$. Therefore, the distance between the foci is $2\cdot 12\sqrt{2}=24\sqrt{2}$.

15. Since the center occurs when $x=2$, the left half satisfies $\frac{(x-2)^2}{3}=\frac{y^2}{5}-1=\frac{y^2-5}{5}$

$$\Rightarrow (x-2)^2=\frac{3y^2-15}{5}\Rightarrow x-2=-\sqrt{\frac{3y^2-15}{5}}\Rightarrow x=2-\sqrt{\frac{3y^2-15}{5}}=2-\sqrt{\frac{1}{5}\left(3y^2-15\right)}.$$

16. In a parabola, any point's distance to the focus is the same as it's distance to the directrix, so the distance is also $\sqrt{317}$.

17. $0=4x^2-y^2-4x+3y-26=4\left(x-\tfrac{1}{2}\right)^2-\left(y-\tfrac{3}{2}\right)^2-\frac{99}{4}=0$, so the equation is also

$$\frac{\left(x-\tfrac{1}{2}\right)^2}{\tfrac{99}{16}}-\frac{\left(y-\tfrac{3}{2}\right)^2}{\tfrac{99}{4}}=1,$$ which makes the slopes of the asymptotes $\pm\frac{\sqrt{\tfrac{99}{4}}}{\sqrt{\tfrac{99}{16}}}=\pm 2$.

18. The angle of rotation θ of the ellipse satisfies $\tan 2\theta = \frac{\sqrt{3}}{2-1} = \sqrt{3} \Rightarrow \theta = 30^\circ$, so the formulas $x = \frac{\sqrt{3}x' - y'}{2}$ and $y = \frac{x' + \sqrt{3}y'}{2}$ will, when substituted, rotate the conic back to having horizontal and vertical axes. Therefore, $2\left(\frac{\sqrt{3}x' - y'}{2}\right)^2$

$$+\sqrt{3}\left(\frac{\sqrt{3}x' - y'}{2} \cdot \frac{x' + \sqrt{3}y'}{2}\right) + \left(\frac{x' + \sqrt{3}y'}{2}\right)^2 - 10 = 0 \Rightarrow \frac{3(x')^2 - 2\sqrt{3}x'y' + (y')^2}{2}$$

$$+\frac{3(x')^2 + 2\sqrt{3}x'y' - 3(y')^2}{4} + \frac{(x')^2 + 2\sqrt{3}x'y' + 3(y')^2}{4} = 10 \Rightarrow \frac{5(x')^2}{2} + \frac{(y')^2}{2} = 10$$

$= \frac{(x')^2}{4} + \frac{(y')^2}{20} = 1$, so the enclosed area is $\pi ab = \pi \cdot 2 \cdot 2\sqrt{5} = 4\pi\sqrt{5}$.

19. The distance between $(2,2)$ and either point is $\sqrt{5}$, and the distance between the other two points is $\sqrt{2}$, so the triangle made with these vertices is isosceles. Drawing the line $y = x$ creates a bisector of the odd side and an altitude of the triangle, and the center of the circle must lie on that line. Therefore, $(x-1)^2 + (x-0)^2 = 2(x-2)^2 \Rightarrow -2x + 1 = -8x + 8 \Rightarrow 6x = 7 \Rightarrow x = \frac{7}{6}$, so the center is at the point $\left(\frac{7}{6}, \frac{7}{6}\right)$, making the radius length $r = \sqrt{\left(1/6\right)^2 + \left(7/6\right)^2} = \sqrt{\frac{50}{36}} = \frac{5\sqrt{2}}{6}$.

20. $0 = 9x^2 - 16y^2 - 18x + 96y - 279 = 9(x-1)^2 - 16(y-3)^2 - 144 \Rightarrow \frac{(x-1)^2}{16} - \frac{(y-3)^2}{9} = 1$ and the conjugate hyperbola switches the two axes while retaining the asymptotes, so its equation would be $\frac{(y-3)^2}{9} - \frac{(x-1)^2}{16} = 1$.

21. $x^2 + y^2 + z^2 - 4x - 2y + 2z \leq 10 \Rightarrow (x-2)^2 + (y-1)^2 + (z+1)^2 \leq 16$, and the solid hemisphere's surface is the solid disk base plus half the surface area of the sphere, which is $\pi(4)^2 + 2\pi(4)^2 = 48\pi$.

22. The inflection point of the logistic equation occurs at half the carrying capacity, which is half the numerator, or $5/2$, so this is the y-coordinate of the inflection point. Plugging this in for y would make $8e^{-3x} = 1 \Rightarrow e^{-3x} = \frac{1}{8} \Rightarrow -3x = \ln\frac{1}{8}$

$\Rightarrow x = -\frac{1}{3}\ln\frac{1}{8} = \ln\left(\frac{1}{8}\right)^{-1/3} = \ln 2$, so the point is $\left(\ln 2, \frac{5}{2}\right)$.

23. $0 = 4y^2 - 12xy + 10x^2 + 2x + 1 = (2y - 3x)^2 + (x+1)^2$, and the right-hand side is a nonnegative quantity, so both terms must equal 0, making this graph the single point $\left(-1, -3/2\right)$.

24. This circle has a diameter of 8, so the circumference is 8π.

25. The given point is on the hyperbola, so by definition the positive difference between these distance equals $2a$. Since the equation can also be written in the form $\frac{x^2}{36} - \frac{y^2}{64} = 1$, this difference is $2\sqrt{36} = 12$.

26. The formula for the focal width is $\frac{2b^2}{a}$, and since $a = 8$ and $b = 5$, the focal width is $\frac{2 \cdot 5^2}{8} = \frac{25}{4}$.

27. The St. Louis Arch is a catenary, which is the shape a chain hanging from two fixed points creates (it looks like a parabola, but it is not).

28. $r = \frac{1}{1 - \cos\theta} \Rightarrow r - r\cos\theta = 1 \Rightarrow \sqrt{x^2 + y^2} = x + 1 \Rightarrow x^2 + y^2 = x^2 + 2x + 1 \Rightarrow x = \frac{1}{2}y^2 - \frac{1}{2}$, and since the focal width of a parabola is the reciprocal of the coefficient of the squared term, the focal width for this parabola is 2.

29. $f(x) = \frac{1}{x^2 - 2|x| + 2} = \frac{1}{(|x| - 1)^2 + 1}$, which is largest when the denominator is smallest. The smallest the squared term can be is 0, so the smallest the denominator can be is 1, making the largest the fraction can be also 1.

30. Based on the values of the eccentricities, the six conic sections are, in order of given eccentricities, circle, parabola, parabola, ellipse, hyperbola, and ellipse. Only the circle and ellipses are closed figures, so the probability that she drew an ellipse is $2/3$.