Answers:

1. D
2. B
3. B
4. D
5. C
6. A
7. A
8. D
9. C
10. D
11. A
12. E
13. E
14. B
15. A
16. D
17. C
18. D
19. B
20. B
21. C
22. A
23. B
24. B
25. C
26. C
27. B
28. A
29. C
30. D

Solutions:

1. The unused portion of a block is $12^2 - \pi(6)^2 = 144 - 36\pi$, so the total you number of pieces of scraps you would need is $\left\lceil \frac{36\pi}{144-36\pi} \right\rceil = \left\lceil \frac{\pi}{4-\pi} \right\rceil = 4$.

2. The sum is $-\frac{0}{1} = 0$ and the product is $\frac{24}{1} = 24$. $|0-24| = 24$

3. $\frac{3.75}{210} = \frac{x}{30} \Rightarrow x = \frac{3.75}{7} = 0.535...$, so to the nearest cent, the cost is $0.54.

4. $99T\left(\frac{91Z}{11T}\right)\left(\frac{3Q}{7Z}\right)\left(\frac{1\sec}{39Q}\right) = 9\sec$ in one direction, so the total time is 18 sec.

5. Since the test is accurate 100% of the time, the probability of having purpleitis is 100%.

6. Using the method of finite differences, the first-order differences are 29, 53, 83, and 119; the second-order differences are 24, 30, and 36; and the third-order differences are 6 and 6. Therefore, the polynomial could possibly have degree 3.

7. $F(10)^2 = \frac{F}{4}(d+10)^2 \Rightarrow d + 10 = 20 \Rightarrow d = 10$

8. $V = \frac{10}{3}\left(40 + 20 + \sqrt{40 \cdot 20}\right) = 200 + \frac{200\sqrt{2}}{3}$

9. $ABC_{16} = 10(16)^2 + 11(16) + 12 = 2748$, so $2748_{16} - ABC_{16} = 1C8C_{16}$

10. $2(\$2.04 + \$2.10 + \$2.16 + ... + \$2.58) = 2(\$23.10) = \$46.20$

11. $\frac{13}{15} - \frac{5}{15} = \frac{8}{15} = 0.533...$, which, to the nearest percentage, is 53%

12. Call the fourth point (a,b). Then, using the shoelace method,

$$\begin{array}{r|cc|l} & 1 & 1 & \\ 2 & 2 & 7 & 7 \\ 35 & 5 & 7 & 14 \\ 7a & a & b & 5b \\ b & 1 & 1 & a \\ 37+7a+b & & & 21+5b+a \end{array} \Rightarrow 14 = A = \frac{1}{2}\left|(37+7a+b)-(21+5b+a)\right| \Rightarrow 28 = |16+6a-4b|$$

$\Rightarrow 14 = |8+3a-2b| \Rightarrow 3a-2b=6$ or $3a-2b=-22$. While not all points on these two lines will give non-degenerate quadrilaterals, there are an infinite number of lattice points on these lines that do. Therefore, there are an infinite number of possible positions.

13. $12(.9(+5)+.1(-1))+12(.5(+5)+.5(-1))+6(.1(+5)+.9(-1))=74.4$

14. The rooks all need to be in different columns, so there are 8 positions to place the first rook. Then there are 7 positions in the second column to place the second rook, 6 positions in the third column to place the third rook, etc., until there is 1 position in the last column to place the last rook. $8!=40320$

15. $2^{10}=1024$, $10^3=1000$, $3^6=729$, and $\binom{14}{4}=\frac{14\cdot 13\cdot 12\cdot 11}{4\cdot 3\cdot 2\cdot 1}=1001$, so of those choices, the first one is largest. For C, the volume is $\pi(6)^2(9)=324\pi \approx 1017.87$, so choice A is still the largest.

16. The sought area is $\frac{(2r)^2\sqrt{3}}{4}-3\cdot\frac{1}{6}\pi r^2=\left(\sqrt{3}-\frac{\pi}{2}\right)r^2$

17. $2T=Tr^2 \Rightarrow r^2=2 \Rightarrow r=\sqrt{2}$, so the number of transistors increases in one year by a factor of $\sqrt{2}$.

18. The minimum number of divisions is 11 (imagine 10 parallel lines), and the maximum number of divisions is $\frac{10\cdot 11}{2}+1=56$. The positive difference in those numbers is $56-11=45$.

19. Imagine the road as being on the x-axis and the vertex of the parabola at $(0,12)$ with zeros of 4 and -4. The equation of the parabola would then be

$y=-\frac{3}{4}(x-4)(x+4)$, and if the train centered up, its height could only be as tall as the y-coordinate at $x=3$ (since the train goes out 3 in both directions).

$y=-\frac{3}{4}(3-4)(3+4)=\frac{21}{4}=5.25$

20. The diameter would be the diagonal of the square, so the area enclosed by the square is $\frac{1}{2}(4)^2=8$.

21. The only shapes that can tile are ones with interior angles whose measure is a factor of 360 (that way the corners meet without leaving any gaps). For a triangle, square, pentagon, and hexagon, the interior angles are 60°, 90°, 108°, and 120°. Of those, only the pentagon's interior angle measure is not a factor of 360.

22. For dollar amounts divisible by 10,000, the take-home amount can be modeled by the function $T(x)=(100000+10000x)(1-.05x)=100000+5000x-500x^2$. The vertex of this parabola is at the point $(5,112500)$, so among those points, 5 additional \$10,000 amounts would maximize the take-home pay. Now, since for any interval of the form $[100000+10000k,109999.99+10000+k]$, the tax rate is the same, you would take home more money for dollar amounts at the right ends of these intervals. We must check the dollar amounts right before 140,000, 150,000, 160,000, and 170,000. Rounded to the nearest cent:

for \$139,999.99, the take-home pay is $(\$139,999.99)(.85)=\$118,999.99$.

for \$149,999.99, the take-home pay is $(\$149,999.99)(.8)=\$119,999.99$.

for \$159,999.99, the take-home pay is $(\$159,999.99)(.75)=\$119,999.99$

for \$169,999.99, the take-home pay is $(\$169,999.99)(.7)=\$118,999.99$.

Since the problem stipulates it is the least amount delivering the most take-home pay, the answer is \$149,999.99. After all, working for that extra \$10,000 to get to \$159,999.99 is fruitless since it would all be paid in taxes!

23. $\frac{{}_{20}P_5}{{}_{26}P_5}=\frac{20\cdot 19\cdot 18\cdot 17\cdot 16}{26\cdot 25\cdot 24\cdot 23\cdot 22}=\frac{3876}{16445}$

24. In 60 minutes, you would make 120 rotations, and thus you would travel 120 circumferences. Therefore, you traveled $120(5\pi)=600\pi$.

25. $\frac{1}{1{,}000{,}000}(60{,}000)(5)(\$200)=\$60$

26. $\frac{2+x/20}{12+x}=\frac{1}{10}\Rightarrow 20+\frac{1}{2}x=12+x\Rightarrow 8=\frac{1}{2}x\Rightarrow x=16$

27. Let $m=$ my current age, and let $s=$ my sister's current age. Therefore, $m=24$ and $2(s-7)=m+4=24+4=28\Rightarrow s-7=14\Rightarrow s=21$.

28. A current computer with 20 memory cells can represent 2^{20} total objects. If x memory cells were used in a quantum computer, the number of total objects that could be represented would be 3^x, so we need the smallest integer x satisfying the inequality $3^x>2^{20}$, implying $x>20\log_2 3$. Since $2^{20}=1048576$, $3^{12}=531441$, and $3^{13}=1594323$, the smallest whole number value for x is 13.

29. The asteroid gains 250 miles on the Earth every hour, so it would take $\frac{10000}{250}=40$ hours for the two objects to collide.

30. It would take Captain Ahab $\frac{2(68)}{8}=17\text{ seconds}$ to row across and back, so if Captain Ahab rowed upstream, he would gain 3 m each second. The river would push Captain Ahab back downstream $17\text{ seconds}\cdot 5\text{ meters/seconds}=85\text{ meters}$, so Captain Ahab should row for $85/3\text{ seconds}$.