

For all questions, answer choice "E) NOTA" means none of the above answers is correct.

1. What is the measure of the acute angle formed by the vectors  $\langle 3,4,5 \rangle$  and  $\langle -1,2,2 \rangle$ ?

- A)  $\frac{\sqrt{2}}{2}$       B)  $\frac{\sqrt{3}}{2}$       C)  $30^\circ$       D)  $45^\circ$       E) NOTA

2. What is the smaller value of the two slopes of the asymptotes of the graph of  $x^2 + 2xy - y^2 = 1$ ?

- A)  $1 + \sqrt{2}$       B)  $1 - \sqrt{2}$       C)  $\sqrt{2} - 1$       D)  $-\sqrt{2} - 1$       E) NOTA

3. What is the shortest distance between two points on the graphs of  $y = \sqrt{-4x - x^2}$  and  $y = 11 - 2x$ , one point on each graph?

- A)  $\frac{11\sqrt{5}}{10}$       B) 5      C)  $3\sqrt{5}$       D)  $\frac{11\sqrt{5} - 20}{10}$       E) NOTA

4. The vertices of a pentagonal region are, in order,  $(1,8)$ ,  $(3,4)$ ,  $(-1,-2)$ ,  $(0,4)$ , and  $(-7,3)$ . What is the area enclosed by this region?

- A) 28.5      B) 36.5      C) 49      D) 24.5      E) NOTA

5. The points  $(2,1,3)$ ,  $(-1,2,1)$ , and  $(4,0,5)$  define a plane. Which of the following points is also on the plane?

- A)  $(92,5,2)$       B)  $(213,4,3)$       C)  $(-11,1,5)$       D)  $(-156,-2,9)$       E) NOTA

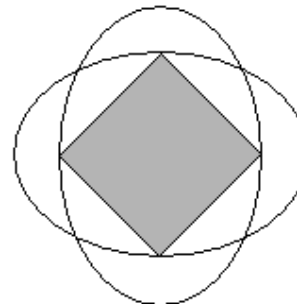
6. What is the focus of the parabola defined by the equations  $x = 8t^2 + 6$  and  $y = 4t - 2$ ?

- A)  $(6.5,-2)$       B)  $(5.5,-2)$       C)  $(-2.5,6)$       D)  $(-1.5,6)$       E) NOTA

7. The graph of  $y = \sqrt{3 + 2x - x^2} + 2$  is reflected across the  $x$ -axis, then rotated  $90^\circ$  clockwise around the point  $(-3,-4)$ . What is the largest  $y$ -value of a point on the resulting figure?

- A)  $-2$       B)  $-4$       C)  $-6$       D)  $-8$       E) NOTA

8. The figure to the right shows two ellipses whose major axes are perpendicular to each other. Each ellipse passes through the other ellipse's foci (which form a square when connected, as shown). Given that the square encloses an area of 16, what is the area enclosed by one of the ellipses?



- A)  $4\pi\sqrt{2}$     B)  $8\pi\sqrt{2}$     C)  $16\pi$     D)  $16\pi\sqrt{2}$     E) NOTA

9. The slope of the line drawn tangent to the curve  $y = \sin x$  at the point where  $x = a$  is equal to  $\cos a$ . Use this fact to find the value of  $x$  on the graph of  $y = \sin x$  for which the distance between the graphs of  $y = \sin x$ , restricted to the domain  $[0, \pi]$ , and the line  $x + 2y = 6$  is minimized.

- A)  $\frac{\pi}{3}$     B)  $\frac{\pi}{2}$     C)  $\frac{2\pi}{3}$     D)  $\frac{5\pi}{6}$     E) NOTA

10. Which of the following transformation matrices  $A$  rotates the point  $(x, y)$  counterclockwise around the origin  $90^\circ$ , then projects the resulting point onto the  $x$ -axis? Note that transformations occur by  $A \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$ .

- A)  $\begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}$     B)  $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$     C)  $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$     D)  $\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$     E) NOTA

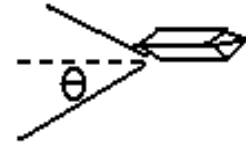
11. Find the length of the shortest path that can be drawn from the point  $(6, 3)$  to the point  $(2, 8)$  such that the path touches both the  $x$ -axis and the  $y$ -axis once.

- A)  $3\sqrt{5} + 2\sqrt{17}$     B)  $\sqrt{149}$     C) 14    D)  $\sqrt{185}$     E) NOTA

12. In the Cartesian coordinate system, consider the fixed points  $c_1 = (-4, 0)$  and  $c_2 = (2, 0)$ . The measure of the acute angle formed by the  $x$ -axis and the segment drawn from  $c_1$  to some other point  $P$  is  $30^\circ$ . The measure of the acute angle formed by the  $x$ -axis and the segment drawn from  $c_2$  to  $P$  is  $60^\circ$ . If  $P = (m, n)$ , what is the sum of all possible positive products  $mn$ ?

- A)  $\frac{57\sqrt{3}}{4}$     B)  $15\sqrt{3}$     C)  $\frac{63\sqrt{3}}{4}$     D)  $\frac{33\sqrt{3}}{2}$     E) NOTA

13. The waves that propagate from the sides of a boat and the boat's axis of symmetry form an acute angle  $\theta$  (as shown in the diagram to the right).  $\tan \theta$  is inversely proportional to the boat's speed, in miles/hour. If  $\sin \theta = \frac{1}{2}$  when the boat is traveling at 15 miles/hour, what is  $\cos \theta$  when the boat is traveling at 20 miles/hour?



- A)  $\frac{4}{5}$       B)  $\frac{\sqrt{51}}{17}$       C)  $\frac{\sqrt{57}}{19}$       D)  $\frac{\sqrt{3}}{4}$       E) NOTA

14. If  $\vec{u} = \langle 6, 4 \rangle$  and  $\vec{v} = \langle 4, 7 \rangle$ , find the vector that is the projection of  $\vec{u}$  onto  $\vec{v}$ .

- A)  $\left\langle \frac{8}{5}, \frac{14}{5} \right\rangle$       B)  $\left\langle \frac{12}{5}, \frac{21}{5} \right\rangle$       C)  $\left\langle \frac{16}{5}, \frac{28}{5} \right\rangle$       D)  $\left\langle \frac{28}{9}, \frac{49}{9} \right\rangle$       E) NOTA

15. How many petals are on the graph of  $r = 2\sin 9\theta$ ?

- A) 4      B) 9      C) 18      D) 36      E) NOTA

16. Determine the positive value of  $k$  such that the lines  $3kx + 4y = 18$  and  $2x - 5\sqrt{k}y = 3$  are perpendicular.

- A)  $\frac{10}{3}$       B)  $\frac{100}{9}$       C)  $\frac{3}{10}$       D)  $\frac{9}{100}$       E) NOTA

17. A triangle has vertices at the points  $(1,3)$ ,  $(4,5)$ , and  $(7,1)$ . What is the length of the longest altitude of the triangle?

- A)  $\frac{9\sqrt{10}}{20}$       B)  $\frac{9\sqrt{10}}{10}$       C)  $\frac{9\sqrt{13}}{13}$       D)  $\frac{18\sqrt{13}}{13}$       E) NOTA

18. The set of all points in space equidistant from the points  $(2,3,2)$  and  $(-4,1,4)$  is given by the equation  $x + By + Cz = D$  for real values  $B$ ,  $C$ , and  $D$ . What is the value of  $B + C + D$ ?

- A)  $-\frac{2}{3}$       B)  $-\frac{4}{3}$       C)  $-\frac{5}{3}$       D)  $-2$       E) NOTA

19. Determine the area enclosed by the polar equation  $r = 6\sin \theta - 2\cos \theta$ .

- A)  $8\pi$       B)  $10\pi$       C)  $12\pi$       D)  $16\pi$       E) NOTA

20. The perpendicular bisector of the line segment with endpoints  $(2,3,2)$  and  $(-4,1,4)$  passes through the point  $(-3,6,1)$  and has equation of the form  $\frac{x+3}{a} = \frac{y-6}{b} = \frac{z-1}{c}$ , where  $a$ ,  $b$ , and  $c$  are relatively prime integers with  $a > 0$ . Evaluate  $abc - (a+b+c)$ .

- A)  $-1$       B)  $-2$       C)  $-3$       D)  $-4$       E) NOTA

21. Determine the length of the latus rectum of the conic section given by  $r = \frac{2}{3-3\sin\theta}$ .

- A)  $\frac{1}{3}$       B)  $\frac{2}{3}$       C)  $\frac{4}{3}$       D)  $\frac{8}{3}$       E) NOTA

22. The foci of an ellipse are located at the points  $(2,4)$  and  $(2,-2)$ . The point  $(4,2)$  lies on the ellipse. If  $a$  and  $b$  represent the lengths of the semi-major and semi-minor axes, respectively, what is the value of  $(ab)^2$ ?

- A)  $26+10\sqrt{10}$     B)  $6+10\sqrt{10}$     C)  $68+22\sqrt{10}$     D)  $6+22\sqrt{10}$     E) NOTA

23. The conic section given by  $4x^2 - 24x - 2y^2 + 8y + 44 = 0$  has eccentricity  $\varepsilon$ . What is the smallest positive degree value of  $a$  such that  $2\sin a = \varepsilon$ ?

- A)  $30^\circ$       B)  $45^\circ$       C)  $60^\circ$       D)  $90^\circ$       E) NOTA

24. Consider the lines  $2x - y = -1$  and  $x - 2y = -5$ . The circle of radius 2 that lies completely in the first quadrant and is tangent to both lines has center at the point  $(a,b)$ . Evaluate  $ab$ .

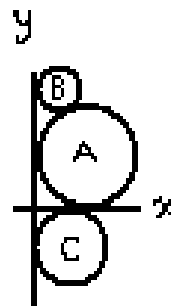
- A)  $10+14\sqrt{5}$     B)  $13+8\sqrt{5}$     C)  $18+10\sqrt{5}$     D)  $23+8\sqrt{5}$     E) NOTA

25. Circle A with radius 4 is tangent to both the positive  $x$ - and  $y$ -axes.

Circle B with radius 1 is tangent to the positive  $y$ -axis and

externally tangent to circle A. Circle C is tangent to the positive  $x$ -axis and negative  $y$ -axis. A line passes through the centers of

circles A, B, and C. What is the radius of circle C? Figure not drawn to scale.



- A) 16      B) 18      C) 24      D) 28      E) NOTA

26. Define a Chelsea circle to be a circle that is tangent to at least one of the  $x$ - or  $y$ -axes.

Consider circle  $O$ , which has the form  $x^2 + cx + y^2 + dy = 0$ , where  $c$  and  $d$  are positive real numbers. Which of the following is true?

I) If  $c = d$ , then circle  $O$  is a Chelsea circle.

II) There exist distinct values for  $c$  and  $d$  such that circle  $O$  is a Chelsea circle.

A) neither is true      B) I only      C) II only      D) I & II      E) NOTA

27. Consider the piecewise function  $f(x) = \begin{cases} x^2 - a, & \text{if } x < 3 \\ b\sqrt{x-2} + a, & \text{if } 3 \leq x < 6 \\ 2x + b, & \text{if } x \geq 6 \end{cases}$ . Given that  $f$  is

continuous, evaluate  $f(1) + f(3) + f(7)$ .

A) 36      B) 39      C) 42      D) 45      E) NOTA

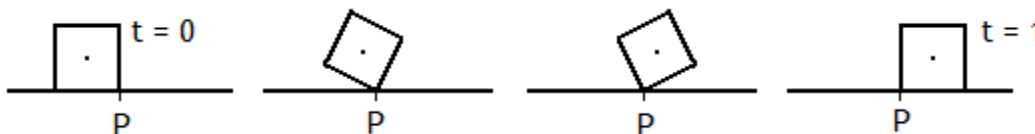
28. Determine the distance between the parallel lines  $4x - 3y = 9$  and  $4x - 3y = 18$ .

A) 1.6      B) 1.7      C) 1.8      D) 2      E) NOTA

29. Determine the product of the slopes of the asymptotes of the hyperbola with equation  $9x^2 - 18x - 4y^2 + 16y - 43 = 0$ .

A)  $-\frac{9}{4}$       B)  $-\frac{4}{9}$       C)  $-\frac{81}{16}$       D)  $-\frac{16}{81}$       E) NOTA

30. A square of side length 2 rotates about a pivot point (labeled P). Which set of equations models the motion of the center of the square on the interval  $[0, 1]$ ? At  $t = 0$  the center of the square is at the point  $(0, 1)$ , and the square completes one  $90^\circ$  turn at  $t = 1$ .



A)  $\begin{cases} x = 1 - \sqrt{2} \cos(\pi/4(2t-1)) \\ y = \sqrt{2} \sin(\pi/4(2t-1)) \end{cases}$       B)  $\begin{cases} x = 1 - \sqrt{2} \sin(\pi/4(2t-1)) \\ y = \sqrt{2} \cos(\pi/4(2t-1)) \end{cases}$

C)  $\begin{cases} x = 1 - \sqrt{2} \sin(\pi/4(2t+1)) \\ y = \sqrt{2} \cos(\pi/4(2t+1)) \end{cases}$       D)  $\begin{cases} x = 1 - \sqrt{2} \cos(\pi/4(2t+1)) \\ y = \sqrt{2} \sin(\pi/4(2t+1)) \end{cases}$

E) NOTA