

For all questions, answer choice "E) NOTA" means none of the above answers is correct.

1. The element Mahonium is highly unstable and decays exponentially. If a 16-gram sample has a half-life of 1 minute, what is its decay constant, in 1/seconds?

- A) $-\frac{\ln 2}{60}$ B) $-\frac{\ln 4}{90}$ C) $-\ln 2$ D) $-\frac{\ln 4}{3}$ E) NOTA

2. Sally is taking her third grade class on a field trip to the science museum. Tickets to the museum cost \$6 for each student and \$10 for each chaperone, and the total cost for admission for Sally's class and chaperones, 26 people total, is \$168. The next week, Sally takes the same group to a movie theater, where student tickets cost \$7 each and chaperone tickets cost \$12 each. What is the total cost of admission for the class to the movie theater?

- A) \$200 B) \$197 C) \$199 D) \$196 E) NOTA

3. The SS Anne sets sail from Vermillion City with a bearing of N 50° E and sails a distance of 35 km to point A. It then turns its bearing to S 20° E and sails an additional 35 km to point B. What is the bearing that the SS Anne must sail in order to return to Vermillion City from point B?

- A) N 55° W B) N 50° W C) N 75° W D) N 70° W E) NOTA

4. Which of the following is equivalent to $\sin x + \sin 5x$ for all real values of x ?

- A) $2\sin 3x \sin 2x$ B) $2\cos 3x \cos 2x$ C) $2\cos 3x \sin 2x$ D) $2\sin 3x \cos 2x$ E) NOTA

5. Find the volume of the tetrahedron with vertices at the points $(0,0,0)$, $(1,1,2)$, $(-3,1,5)$, and $(7,2,4)$.

- A) 7.5 B) 5 C) 2.5 D) 1 E) NOTA

6. The region of the xy -plane described by the inequality $3x^2 - 6x + 3y^2 - 18y + 3 \leq 0$ is rotated about the line $x = 1$ to form a solid figure. If A and V represent the surface area and the volume of the figure, respectively, what is the value of $\frac{A}{V}$?

- A) $\frac{3}{4}$ B) 1 C) $\frac{4}{7}$ D) $\frac{3}{7}$ E) NOTA

7. Find the product of the solutions to the equation $\log_{3x} x = 2$.

- A) $\frac{1}{9}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) 0 E) NOTA

8. In a factory, machines A, B, and C produce both normal and holographic Pokémon cards, but no other type of cards. Of their production, machines A, B, and C produce 1%, 4%, and 3% holographic cards, respectively. Machines A, B, and C produce 30%, 25%, and 45% of the total output of cards, respectively. Madhukar randomly selects one card from the factory's output and is disappointed to discover that it is a normal Pokémon card. Find the probability that machine B produced the normal card Madhukar selected.

- A) $\frac{18}{59}$ B) $\frac{20}{53}$ C) $\frac{160}{649}$ D) $\frac{291}{649}$ E) NOTA

9. Find the product of the solutions for x in the equation $(x^{\frac{1}{x}})(x) = (x^x)$. All quantities in parentheses in the equation must be real.

- A) -1 B) $\frac{1+\sqrt{5}}{2}$ C) 1 D) $-\frac{1+\sqrt{5}}{2}$ E) NOTA

10. A new game is being proposed for Bonnie's Big Bucks Casino. For this game, a player will pay P dollars to play, and the rules are as follows: On his first turn, the player rolls a standard, fair, six-sided die, and if the die reads 1, the player wins \$1 and the game ends; otherwise, he continues to the second turn. On his second turn, the player rolls the same die, and if the die reads 2, the player wins \$2 and the game ends; otherwise, he continues to the third turn. On his third turn, the player rolls the same die, and if the die reads 3, the player wins \$3 and the game ends; otherwise, the player wins \$1 and the game ends. Find the lowest value for P , rounded up to the nearest half-dollar that allows the expected monetary return of this game for the casino to be positive.

- A) \$3 B) \$2.50 C) \$2 D) \$1.50 E) NOTA

11. Find the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} f(n)}{5^n}$, where $f(n) = \begin{cases} 1, & \text{if } n \text{ is odd} \\ 2, & \text{if } n \text{ is even} \end{cases}$.

- A) $\frac{1}{16}$ B) $\frac{1}{12}$ C) $\frac{1}{10}$ D) $\frac{1}{6}$ E) NOTA

12. In Ted's middle school, there are 2011 hallway lockers numbered as integers from 1 to 2011, inclusive, all of which are closed. Being very bored, Ted opens every locker. Then Ted closes all the even numbered lockers. Then Ted changes the position (open or closed) for every locker numbered with a multiple of 3. Then Ted changes the position for every locker numbered with a multiple of 4. This pattern continues until Ted changes the position of only the 2011th locker. After Ted has finished, he decides to find the sum of the numbers of lockers left open. What is the value of this sum?

- A) 29370 B) 31395 C) 1991671 D) 1993696 E) NOTA

13. What is the diameter, in feet, of a pulley which is being drive at 5 revolutions/second by a belt moving at 40 feet/second?

- A) 8 B) 16 C) $\frac{8}{\pi}$ D) $\frac{16}{\pi}$ E) NOTA

14. What is the units' digit of the quantity $9^{412} + 16^8 + 5^{120}$?

- A) 0 B) 1 C) 2 D) 3 E) NOTA

15. Find the domain of the function $f(x) = \log_4(\log_{30}(\log_{1991} x))$

- A) $x > 0$ B) $x > 1991$ C) $x > 1991^{30}$ D) $x > 1991^{30^4}$ E) NOTA

16. Line 1 has the equation given by $y = 2x - 3$, while Line 2 has the equation given by $y = 3x + 5$. What is the cosine of the smaller angle made between these two lines?

- A) $\frac{17\sqrt{105}}{210}$ B) $\frac{7\sqrt{2}}{10}$ C) $\frac{\sqrt{5}}{10}$ D) $\frac{15\sqrt{2}}{38}$ E) NOTA

17. Find the sum of the solutions to the equation $\log_{\sin\theta} \cos\theta + \log_{\cos\theta} \tan\theta = 1$ on the interval $0 \leq \theta < 2\pi$.

- A) 4π B) $\frac{3\pi}{2}$ C) $\frac{2\pi}{3}$ D) $\frac{\pi}{4}$ E) NOTA

18. Find the sum of the solutions to the equation $\log x + \log(x+2) = \frac{3}{2} \log 4$.

- A) -4 B) -2 C) 4 D) 2 E) NOTA

19. What is the value of z in the system of equations $\begin{cases} 19x - 12y + 15z = 90 \\ 27x + 13y + 75z = 72. \\ 3x - 51y + 32z = 61 \end{cases}$

- A) $\frac{37687}{71007}$ B) $-\frac{4593}{12007}$ C) $-\frac{25433}{67007}$ D) $\frac{14123}{37007}$ E) NOTA

20. Given that $x - y = 2$ and $x^2 - y^2 = 12$, find the value of the product xy .

- A) 4 B) 8 C) 6 D) 10 E) NOTA

21. Paul is shopping at the White House gift shop. He wants to buy a coffee mug with a sticker price of \$30 and a poster with a sticker price of \$15. Paul uses a 10% off coupon for the mug, then gets a 20% discount on the total purchase price before tax with his school ID. How much does he pay, assuming he has to pay the local sales tax of 5.75%? Round to the nearest cent.

- A) \$35.50 B) \$35.51 C) \$35.52 D) \$35.53 E) NOTA

22. In any given discussion, there is a $\frac{4}{5}$ probability that Stacy resorts to fallacy, a $\frac{1}{5}$ probability that Nancy resorts to fallacy, and a $\frac{7}{16}$ probability that Lauren resorts to fallacy. Assuming independence of their fallacious tendencies, what is the probability that exactly two of them resort to fallacy in a discussion amongst the three of them?

- A) $\frac{31}{80}$ B) $\frac{7}{100}$ C) $\frac{183}{400}$ D) $\frac{77}{200}$ E) NOTA

23. Solve for x : $\begin{vmatrix} 3 & x & x^2 \\ 5 & -2 & 9 \\ -1 & 1 & -3 \end{vmatrix} = 15$

- A) 4 and 2 B) 4 and -2 C) -4 and 2 D) -4 and -2 E) NOTA

24. Mason's love of reading has been increasing lately. On July 1, he read one book. On July 2, he read 2 books. On July 3, he read 4 books. If he keeps reading twice as many books each day as he did on the previous day, how many books will Mason read during the month of July?

- A) $2^{30} - 1$ B) $2^{31} - 1$ C) $2^{32} - 1$ D) $2^{33} - 1$ E) NOTA

25. What is the coefficient of the x^3y^4 -term in the expansion of $\left(2x - \frac{y}{3}\right)^7$?

- A) $\frac{280}{81}$ B) $-\frac{280}{81}$ C) $\frac{8}{81}$ D) $-\frac{8}{81}$ E) NOTA

26. At the imaginary Bluff Park High School, the 5 distinct officer positions of Mu Alpha Theta are selected by randomly drawing cards without replacement from a standard, 52-card deck and noting the number from the card (assume an ace to be 1, jack to be 11, queen to be 12, and king to be 13). The person with the highest number becomes president, and each respective lower total wins the next lower office, which is perceived to be less prestigious. If 8 people are in the running for the five offices, how many possible arrangements are there of people in the 5 offices?

- A) 56 B) 70 C) 1680 D) 6720 E) NOTA

27. Melissa and Anthony are best friends. Each of them has a favorite time which changes every day. Melissa's favorite time is modeled by an arithmetic sequence starting at 12:30 am on January 1 and increasing by 3 minutes every day. Anthony's favorite time is noon on January 1, 10 pm on January 2, and thereafter, on any day, his favorite time is the number halfway between 5 pm and his favorite time two days before. For the time on January 6 that is halfway between their favorite times on that day, what is the measure of the smaller angle between the hour and minute hands of a clock at that time?

- A) 105° B) 75° C) 60° D) 45° E) NOTA

28. Ronald and Jimmy are working together pruning a rose garden. Ronald can prune the garden alone in 8 hours. Working together, they can prune the garden in 5 hours. How many hours would it take Jimmy to prune the garden alone?

- A) 6 B) $\frac{40}{3}$ C) 2 D) $\frac{40}{13}$ E) NOTA

29. What is the remainder when $x^{2011} - 5x^{1991} + 2x^{999} - x^2 + 3x - 1$ is divided by $x + 1$?

- A) -13 B) -1 C) -3 D) -11 E) NOTA

30. Evaluate: $0! + \lim_{x \rightarrow 0} \frac{\tan x}{x}$

- A) 0 B) 1 C) 2 D) undefined E) NOTA