Answers:

1. D
2. E
3. B
4. D
5. A
6. C
7. C
8. C
9. D
10. C
11. C
12. C
13. A
14. D
15. B
16. D
17. C
18. D
19. A
20. B
21. B
22. E
23. A
24. C
25. A
26. B
27. C
28. E
29. C
30. A

Solutions:

1. Since $\angle A$ and $\angle C$ are both supplementary to $\angle B$, $\angle A$ and $\angle C$ must be equal. Thus, $3x+16=5x-24 \Rightarrow 2x=40 \Rightarrow x=20 \Rightarrow m\angle A=m\angle C=3\cdot 20+16=76^\circ$. Therefore, $m\angle B=104^\circ$.

2. Any three points are always coplanar.

3. The midpoint is $\left(\frac{-4+19}{2},\frac{-14-10}{2},\frac{27+3}{2}\right)=(7.5,-12,15)$.

4. An "only if" statement is the same as an "if-then" in the same order. Therefore, the statement is equivalent to "If Mr. Snube is happy, then the weather is not good."

5. On a sphere, the sum of the angles of a triangle must sum to more than 180° and less than 540°, so choice A would not work for a triangle.

6. Since $m\angle ABE=8^\circ$, $m\angle ECD=8^\circ$ also. Therefore, DE is $\frac{8}{360}=\frac{1}{45}$ of the total circumference, so the total circumference is $60\cdot 45=2700\text{ miles}$.

7. Since $a=b$, in line (8), division by 0 is occurring, so that is the error.

8. K itself is false, having a true premise and false conclusion. The converse of K is $q\rightarrow p$, which is a false premise implying a true conclusion, so the converse is true. The inverse of K is $\sim p\rightarrow\sim q$, which is the contrapositive of the converse, thus having the same truth value, so it is true. The contrapositive of K, $\sim q\rightarrow\sim p$, has the same truth value as K, so it is false. Thus, two of the statements are true.

9. Since $m_{AB}=\frac{14-5}{24-12}=\frac{9}{12}=\frac{3}{4}$, $\overline{AB}$ is not the hypotenuse of the triangle. Either $\overline{BC}$ or $\overline{AC}$ is the other leg, which would be perpendicular to $\overline{AB}$. $m_{BC}=\frac{14-k}{24-0}$ $=\frac{14-k}{24}$, so if $\overline{BC}$ is the other leg, then $\frac{14-k}{24}=-\frac{4}{3}\Rightarrow k=46$, but this would make $\overline{AC}$ the hypotenuse, and $m_{AC}=\frac{5-k}{12}=-\frac{41}{12}\neq-\frac{7}{24}$, so this is not possible. Therefore, $\overline{AC}$ is the other leg, making $\frac{5-k}{12}=-\frac{4}{3}\Rightarrow k=21$, which makes $m_{BC}=\frac{14-k}{24}=-\frac{7}{24}$.

Therefore, $\overleftrightarrow{BC}$ goes through the two points $(0,21)$ and $(24,14)$, making its equation $y=-\frac{7}{24}x+21$, and its x-intercept would satisfy $0=-\frac{7}{24}x+21\Rightarrow\frac{7}{24}x=21$ $\Rightarrow x=72$

10. The ball has radius 6, and the first octant wedge contains $\frac{1}{8}$ of the sphere surface plus $\frac{3}{4}$ of the disk with radius 6 as its surface. Thus, the total surface area of the region is $\frac{3}{4}(\pi\cdot 6^2)+\frac{1}{8}(4\pi\cdot 6^2)=27\pi+18\pi=45\pi$.

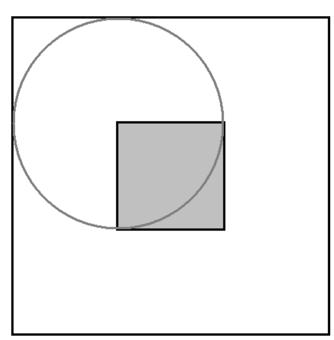
11. $m\angle W=180^\circ-67^\circ-57^\circ=56^\circ$, and the shortest side of the triangle is opposite the smallest angle, which is $\angle W$, thus making the shortest side $\overline{FB}$.

12. Isometries preserve length measurements, and translations, reflections, and rotations all do that. Dilations scale the graphs up or down, so they are not isometries.

13. The circle with larger radius encloses more area, so we just need to compare the radii lengths. Raising both radii to the 30th power, gives measurements of $\left(\sqrt[3]{2}\right)^{30}=2^{10}=1024$ and $\left(\sqrt[10]{10}\right)^{30}=10^3=1000$, so circle J encloses more area.

14. A is angle-angle-angle, which only shows similarity, not congruence. B is side-side-angle, which is not a congruence condition. C does not compare corresponding parts of the two triangles. D is side-angle-side, which is a congruence condition, so this would show the two triangles are congruent.

15. The altitudes are perpendicular to the side lengths, so their slopes are negative reciprocals of the slopes of the side lengths. The three sides' slopes are $\frac{-2-4}{7-1}=\frac{-6}{6}=-1$, $\frac{6-4}{-3-1}=\frac{2}{-4}=-\frac{1}{2}$, and $\frac{-2-6}{7-(-3)}=\frac{-8}{10}=-\frac{4}{5}$, so the slopes of the three altitudes are 1, 2, and $\frac{5}{4}$, and the sum of the slopes is $1+2+\frac{5}{4}=\frac{17}{4}$.

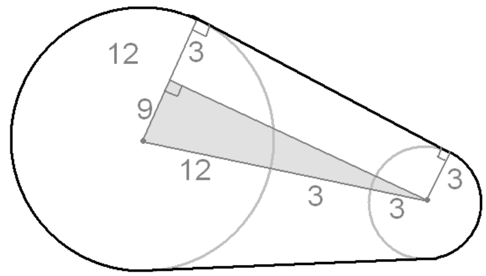


16. Within a single square, the center of the coin must lie inside the inner square with side length 1. Therefore, there is a total area of 64 where the center can land. Since the coin lands entirely on the checkerboard, the

total area in which the center can land is a 22×22 section in the center of the checkerboard. Therefore, the probability of the coin landing inside one of the squares on the checkerboard is $\frac{64}{22^2}=\frac{64}{484}=\frac{16}{121}$.

17. $\angle HMA$ and $\angle TMA$ are the same angle, so they are congruent by the Reflexive Property of Congruence

18. The length of the common external tangent is $\sqrt{18^2-9^2}=\sqrt{324-81}=\sqrt{243}=9\sqrt{3}$, so the shaded triangle is a $30^\circ, 60^\circ, 90^\circ$ right triangle. Therefore, the total length of the pulley is $\frac{2}{3}(2\pi\cdot12)+\frac{1}{3}(2\pi\cdot3)+2(9\sqrt{3})=16\pi+2\pi$ $+18\sqrt{3}=18\pi+18\sqrt{3}$.



19. Alternate interior angles are on opposite sides of the transversal and inside the two lines, so $\angle 6$ and $\angle 3$ satisfy this (lines do not have to be parallel to have alternate interior angles). $\angle 10$ and $\angle 11$ are equivalent angles, $\angle 12$ and $\angle 8$ are same-side interior angles, as are $\angle 6$ and $\angle 11$.

20. The three cities make a 400-500-700 triangle, and $400^2+500^2=160000+250000$ $=410000\neq490000=700^2$, so this is not a right triangle. Since Cincinnati, OH is due west of Washington, DC, Tallahassee, FL could not possibly be due south of Washington, DC (if it was, there would be a right angle at Washington, DC).

21. This revolution results in a cylinder in the middle with radius 7 and height 14, and two cones with radius 7 and height 3. Therefore, the total volume of the solid is $\pi(7)^2(14)+2\left(\frac{1}{3}\pi(7)^2(3)\right)=686\pi+98\pi=784\pi$.

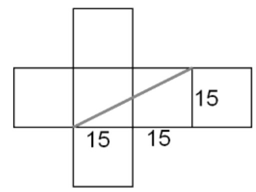
22. The orthocenter, circumcenter, and centroid of a triangle always lie on the Euler line in a triangle, so the triangle is not necessarily any of the given types.

23. If $\overline{HK}$ is the longest side, then it satisfies $\left|\overline{HK}\right|<5+4+10=19$. If $\overline{HK}$ is the shortest side, then it satisfies $\left|\overline{HK}\right|+5+4>10\Rightarrow\left|\overline{HK}\right|>1$. Therefore, $\overline{HK}$ can take on any integer value from 2 to 18, inclusive, which is 17 different values.

24. Considering the diagonals that connect X to consecutive vertices and considering the circle circumscribing the dodecagon, the angle is inscribe and intercepts an arc of measure 30° (since $360/12 = 30$). Since this is an inscribed angle, it has measure half the intercepted arc, or 15°.

25. $V = \frac{6}{3}\left(12 + 27 + \sqrt{12 \cdot 27}\right) = 2(12 + 27 + 18) = 2(57) = 114$

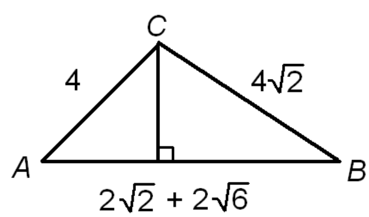
26. Unfolding the box and drawing a straight line from one vertex to the opposite vertex forms a right triangle with legs of length 15 and 30. Therefore, the hypotenuse is the spider's path, which has length $\sqrt{15^2 + 30^2} = \sqrt{225 + 900} = \sqrt{1125} = 15\sqrt{5}$.



27. One pair of opposite sides are parallel and congruent, and they are not the same side (which would create a degenerate quadrilateral), so this must be a parallelogram.

28. Kites, trapezoids, rhombi (if they are squares), and rectangles could all be inscribed in circles.

29. If $\overline{AC}$ is the longest side, then $\left|\overline{AB}\right| = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$. If $\overline{AB}$ is the longest side, then $\left|\overline{AB}\right| = \sqrt{5^2 + 3^2} = \sqrt{25 + 9} = \sqrt{34}$. Therefore, two lengths are possible to make right triangles.



30. Let D be the point on $\overline{AB}$ such that $\left|\overline{AD}\right| = 2\sqrt{2}$ and $\left|\overline{BD}\right| = 2\sqrt{6}$. Dropping an altitude from C to $\overline{AB}$ intercepts $\overline{AB}$ at point E. Checking the Pythagorean Theorem shows that $\left|\overline{CE}\right| = \sqrt{4^2 - \left(2\sqrt{2}\right)^2} = \sqrt{16 - 8} = \sqrt{8} = 2\sqrt{2}$ and $\left|\overline{CE}\right|$ $= \sqrt{\left(4\sqrt{2}\right)^2 - \left(2\sqrt{6}\right)^2} = \sqrt{32 - 24} = \sqrt{8} = 2\sqrt{2}$ if E and D are the same point, and since these lengths are equal, E and D are the same point. Therefore, $m\angle A = 45^\circ$, $m\angle B = 30^\circ$, and $m\angle C = 105^\circ$, meaning $(m\angle A + m\angle C) \cdot m\angle B = (45 + 105) \cdot 30$ $= 150 \cdot 30 = 4500$.