For all questions, answer choice "E) NOTA" means none of the above answers is correct.

1. Let A be the greatest common divisor of 221063 and 218929, and let $B = A + 5$. Find the number of positive integral divisors of B.

A) 2 B) 4 C) 6 D) 8 E) NOTA

2. A perfect number X is an integer greater than 1 for which the sum of its positive integral divisors equals $2X$. Let A and B be the smallest two perfect numbers. Find the number of positive integral divisors of $A + B$.

A) 3 B) 6 C) 2 D) 4 E) NOTA

3. The set $\{\ldots,-3,-2,-1,0,1,2,3,\ldots\}$ is frequently denoted by which of the following?

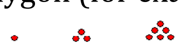
A) $\mathbb{N}$ B) $\mathbb{R}$ C) $\mathbb{Z}$ D) $\mathbb{Q}$ E) NOTA

4. A, B, and C are digits in the range $0-9$. Find the value of $A + B + C$ if the sum of the three-digit number $3A4$ and the four-digit number $1ACB$ equals the four-digit number $1C7C$.

A) 6 B) 8 C) 13 D) 16 E) NOTA

5. Consider the system of equivalences $\begin{cases} x \equiv 1 \pmod{3} \\ x \equiv 2 \pmod{5} \\ x \equiv 3 \pmod{7} \\ x \equiv 4 \pmod{11} \end{cases}$. If A and B are the smallest two positive integral solutions to the system, find the value of $A + B$.

A) 1889 B) 2659 C) 1155 D) 1504 E) NOTA

6. A polygonal number is a positive integer which can be represented as a number of dots in the shape of a particular regular polygon (for example, the following image depicts the first 3 triangular numbers, 1, 3, and 6: ). Find the smallest positive integer greater than 1 which is both a triangular number and a square number.

A) 6 B) 36 C) 25 D) 1225 E) NOTA

7. If the current time is 7:34:52 on a 24-hour clock, find the time on the clock after 35 hours, 56 minutes, and 234 seconds.

A) 18:30:46 B) 19:34:46 C) 18:34:46 D) 19:30:46 E) NOTA

8. Find the 7th element in the 15th row of Pascal's triangle if row 1 is "1 1".

A) 6435 B) 3003 C) 3432 D) 105 E) NOTA

9. If the positive integral divisors of 4096 are written in increasing order $a_1, a_2, \ldots, a_n$, where $a_i < a_{i+1}$ for all integers i, $1 \le i \le n-1$, find the value of $\sum_{i=1}^{n}(-1)^{i+1} a_i$.

A) -1364 B) 2049 C) 0 D) 2731 E) NOTA

10. Let A be the smallest positive integer less than 100 with the greatest number of positive integral divisors, and let B be that number of positive integral divisors. Find the sum of all positive integers which have exactly B positive integral divisors.

A) 306 B) 312 C) 318 D) 330 E) NOTA

11. $x_{10} = 221_a - 101_b$ and $b - a = 8$. Find the minimum value of x.

A) -112 B) -255 C) -63 D) -113 E) NOTA

12. How many integer values of x satisfy $x \equiv 1 \pmod{2}$, $x \equiv 2 \pmod{3}$, and $29 < x < 72$?

A) 7 B) 6 C) 9 D) 8 E) NOTA

13. The Fibonacci numbers are a sequence defined by the recurrence $F_{n+2} = F_{n+1} + F_n$, where $F_1 = F_2 = 1$. Find the value of $F_{-8} + F_8$.

A) 8 B) 42 C) 0 D) 16 E) NOTA

14. If a_n is the number of positive integral divisors of n, find the value of $\sum_{n=1}^{10} a_n$.

A) 27 B) 25 C) 28 D) 26 E) NOTA

15. Find the sum of the digits of the smallest positive integer x such that $260 \cdot x \equiv 1 \pmod{43}$.

A) 10 B) 6 C) 4 D) 1 E) NOTA

16. Find the last two digits of the expanded quantity 2^{96}.

A) 02 B) 96 C) 16 D) 36 E) NOTA

17. If b and c are real numbers such that $f(x) = x^5 + bx^2 + 2x - c$ (i) is divisible by $x+1$, and (ii) leaves a remainder of 12 when divided by $x-2$, find the remainder when $f(x)$ is divided by $x+2$.

A) -12 B) -60 C) -84 D) 12 E) NOTA

18. What is the largest even integer which cannot be expressed in the form $14w + 12x + 24y + 26z$, where w, x, y, and z are nonnegative integers?

A) 56 B) 46 C) 54 D) 44 E) NOTA

19. A Mersenne prime is a prime number of the form $2^p - 1$, where p is prime. Find the first prime number p such that $2^p - 1$ is not a Mersenne prime (i.e., $2^p - 1$ is composite).

A) 13 B) 23 C) 7 D) 11 E) NOTA

20. Find the value of n such that $251_n + 121_{n+1} = 415_n$.

A) 8 B) 6 C) 7 D) 9 E) NOTA

21. If $x = \frac{2011!}{7^n}$, find the smallest positive integer n such that x is not divisible by 10.

A) 2011 B) 334 C) 288 D) 224 E) NOTA

22. If $123A782B$ (where A and B are among the digits $0-9$) is divisible by 2 and 3, how many ordered pairs (A, B) are possible?

A) 15 B) 17 C) 16 D) 19 E) NOTA

23. A certain number A has prime factorization of the form $2^a3^b5^c$, where a, b, and c are positive integers. If $x =$ the number of positive integral divisors of A that are divisible by 2, $y =$ the number of positive integral divisors of A that are divisible by 3, and $z =$ the number of positive integral divisors of A that are divisible by 5, find the value of $x+y+z$.

A) $3abc+2ab+2ac+2bc+a+b+c$ B) $abc+ab+ac+bc+a+b+c+1$
C) $ab+ac+bc+2a+2b+2c+3$ D) $3abc+ab+ac+bc$ E) NOTA

24. If $x_9 = 210102_3$ and $y_8 = 120032_4$, find the sum, considered as base-10, of the digits of x and y.

A) 14 B) 20 C) 31 D) 13 E) NOTA

25. How many positive integers less than 100 are relatively prime with 12?

A) 25 B) 32 C) 17 D) 34 E) NOTA

26. What is the smallest positive integer x satisfying $23^{43} \equiv x \pmod{43}$?

A) 1 B) 23 C) 20 D) 12 E) NOTA

27. The Fibonacci numbers are as defined in question 13. Find the value of $\lim_{n\to\infty} \frac{F_{n+1}}{F_n}$.

A) $\frac{1+\sqrt{5}}{2}$ B) 1.5 C) 1.6 D) $\frac{1+\sqrt{3}}{2}$ E) NOTA

28. The Fermat Polygonal Number Theorem states that every positive integer can be expressed as the sum of at most n n-gonal numbers for all integers $n \geq 3$ (e.g., 3 triangular numbers, 4 square numbers, etc.). For example, 7 can be written as $3+3+1$ if writing triangular numbers, $4+1+1+1$ if writing square numbers, or $5+1+1$ if writing pentagonal numbers (there may be more than one representation for a given value of n, but not for this example). Find the sum of the squares of the numbers of all representations used to express the number 8 as the sum of n n-gonal numbers for all integers n, $3 \leq n \leq 5$. For the example given, the answer would be $3^2+3^2+1^2+4^2+1^2$ $+1^2+1^2+5^2+1^2+1^2 = 65$.

A) 98 B) 60 C) 70 D) 104 E) NOTA

29. Use the ordered pair of non-negative integers whose coordinates have the least sum (excluding the trivial solutions $(1,0)$ and $(3,2)$) to the Pell equation $x^2-2y^2=1$ to approximate $\sqrt{2}$ to five decimal places. We would normally use the equation $x^2-2y^2=0$, but this could be solved for an exact value of $\sqrt{2}$, and there are no non-trivial integers whose quotient is exactly $\sqrt{2}$. Therefore, we perform the calculation similarly, only using the 1 in the equation to get an ordered pair that fits the equation; the 1 is not used in the calculation of $\sqrt{2}$. (Hint: $x, y<25$)

A) 1.41667 B) 1.41333 C) 1.41421 D) 1.41525 E) NOTA

30. Find the number of ordered pairs of positive integers (x, y) satisfying the equation $x^2 y=14400$.

A) 63 B) 28 C) 24 D) 16 E) NOTA