Answers:

0. $\pi+2$
1. 1
2. 6
3. 609
4. 4
5. 41
6. $1-e$
7. $-\frac{15}{4}$
8. 299
9. $\frac{4-9\pi}{16}$
10. $\frac{46}{15}$
11. 2.0883
12. A, B, C, D, E (any order)
13. $-4e^{4}$
14. -4

Solutions:

0. $A=\lim_{x\to\infty}\left(2\tan^{-1}x\right)=2\left(\frac{\pi}{2}\right)=\pi$

$B=\lim_{x\to 2}\frac{x^2+8x-20}{x^3-8}=\lim_{x\to 2}\frac{2x+8}{3x^2}=\frac{2(2)+8}{3(2)^2}=\frac{12}{12}=1$

$C=\lim_{x\to 0^+}x^x=1$

$D=\lim_{x\to\pi}\tan x=0$

$A+B+C+D=\pi+1+1+0=\pi+2$

1. $A=\frac{f(2)-f(1)}{2-1}=\frac{0-(-3)}{1}=3$

$B=f'(1)=4(1)-3=1$

$C=\frac{f(3)-f(2)}{3-2}=\frac{7-0}{1}=7$

$D=f'(2)=4(2)-3=5$

$\frac{A-B}{C-D}=\frac{3-1}{7-5}=\frac{2}{2}=1$

2. $A=\lim_{x\to -1}\frac{x^3+x+2}{2x^4+3x+1}=\lim_{x\to -1}\frac{3x^2+1}{8x^3+3}=\frac{3(-1)^2+1}{8(-1)^3+3}=-\frac{4}{5}$

$f(x)=\frac{x-2}{x+1}\Rightarrow f'(x)=\frac{(x+1)(1)-(x-2)(1)}{(x+1)^2}=3(x+1)^{-2}\Rightarrow f''(x)=-6(x+1)^{-3}$

$B=f''(-3)=-6(-3+1)^{-3}=\frac{3}{4}$

$f(x)=-7x^{3/2}+21x-18\Rightarrow f'(x)=-\frac{21}{2}x^{1/2}+21\Rightarrow f'(x)=0$ when $x=4$

$C=f(4)=-7(4)^{3/2}+21(4)-18=-56+84-18=10$

$f(x)=x^2-2x+2x\ln x\Rightarrow f'(x)=2x-2+2x\left(\frac{1}{x}\right)+2\ln x=2x+2\ln x\Rightarrow f''(x)=2+\frac{2}{x}$

$f''(x)$ changes signs when $x=-1$, but this is not in the domain, so $D=-1$

$ABCD=\left(-\frac{4}{5}\right)\left(\frac{3}{4}\right)(10)(-1)=6$

3. $S=2x+y=2x+\frac{n}{x}\Rightarrow S'=2-\frac{n}{x^2}\Rightarrow S'=0$ when $x=\sqrt{\frac{n}{2}}$ and this value gives a

minimum by sign chart

$$A=2\sqrt{\frac{50}{2}}+\frac{50}{\sqrt{50/2}}=2(5)+10=20$$

$$B=2\sqrt{\frac{128}{2}}+\frac{128}{\sqrt{128/2}}=2(8)+16=32$$

$$C=2\sqrt{\frac{162}{2}}+\frac{162}{\sqrt{162/2}}=2(9)+18=36$$

$$D=2\sqrt{\frac{288}{2}}+\frac{288}{\sqrt{288/2}}=2(12)+24=48$$

$$\frac{\left(A^2+\sqrt{C}\right)D}{B}=\frac{\left(20^2+\sqrt{36}\right)(48)}{32}=\frac{(406)(3)}{2}=609$$

4. $f(x)=(x-1)^2(x+1)$, so the intercepts are at $(-1,0)$ and $(1,0)$

$f'(x)=3x^2-2x-1=(3x+1)(x-1)\Rightarrow f'(x)=0$ when $x=-\frac{1}{3}$ (min) or $x=1$ (max)

relative extrema are at $\left(-\frac{1}{3},\frac{32}{27}\right)$ and $(1,0)$

$f''(x)=6x-2,$ so inflection point occurs when $x=\frac{1}{3}$, and point is $\left(\frac{1}{3},\frac{16}{27}\right)$

$A=-1+1=0,\ B=-\frac{1}{3}+1=\frac{2}{3},\ C=\frac{1}{3}$, and $D=0+0+0+\frac{32}{27}+\frac{16}{27}=\frac{48}{27}=\frac{16}{9}$

$$(4A+3B+3C)\sqrt{D}=\left(4\cdot 0+3\cdot\frac{2}{3}+3\cdot\frac{1}{3}\right)\sqrt{\frac{16}{9}}=3\cdot\frac{4}{3}=4$$

5. From quadratic equation, $s(t)=0$ when $t=\frac{3+\sqrt{41}}{4}\approx 2.350$, so $\lfloor A\rfloor=2$

$s'(t)=-32t+24\Rightarrow s'(t)=0$ when $t=\frac{3}{4}(=B)$

$$C=s'(A)=-32\left(\frac{3+\sqrt{41}}{4}\right)+24=-8\sqrt{41}$$

$$\left\lfloor\lfloor A\rfloor B\right\rfloor\left(\frac{C}{8}\right)^2=\left\lfloor 2\cdot\frac{3}{4}\right\rfloor\left(-\sqrt{41}\right)^2=1\cdot 41=41$$

6. $f(x)=\dfrac{\ln x}{x-1}\Rightarrow f'(x)=\dfrac{(x-1)\cdot\frac{1}{x}-\ln x}{(x-1)^2}$

$$A=f'(e)=\frac{\frac{e-1}{e}-1}{(e-1)^2}=-\frac{1}{e(e-1)^2}$$

$$L(x)=-\frac{1}{e(e-1)^2}(x-e)+\frac{1}{e-1}=-\frac{1}{e(e-1)^2}x+\frac{e}{(e-1)^2}=\frac{-x+e^2}{e(e-1)^2}$$

$$L(x)=0\Rightarrow x=e^2\,(=B)$$

$$C=L(0)=\frac{e^2}{e(e-1)^2}=\frac{e}{(e-1)^2}$$

$$D=L(1)=\frac{-1+e^2}{e(e-1)^2}=\frac{(e-1)(e+1)}{e(e-1)^2}=\frac{e+1}{e(e-1)}$$

$$\frac{A(B+e)}{CD}=\frac{-\frac{1}{e(e-1)^2}\cdot(e^2+e)}{\frac{e}{(e-1)^2}\cdot\frac{e+1}{e(e-1)}}=\frac{-\frac{e+1}{(e-1)^2}}{\frac{e+1}{(e-1)^3}}=-(e-1)=1-e$$

7. $A=\displaystyle\int_1^3\frac{x}{x^2+1}\,dx=\frac{1}{2}\ln\left|x^2+1\right|\Big|_1^3=\frac{1}{2}\ln10-\frac{1}{2}\ln2=\frac{1}{2}\ln5$

$$B=\int_{-2}^1\left(x^3-4x^2+2x-3\right)dx=\left(\frac{1}{4}x^4-\frac{4}{3}x^3+x^2-3x\right)\Big|_{-2}^1$$

$$=\left(\frac{1}{4}-\frac{4}{3}+1-3\right)-\left(4+\frac{32}{3}+4+6\right)=-\frac{111}{4}$$

$$C=\int_{-2}^2\frac{\tan^{-1}x}{1+x^2}\,dx=\frac{1}{2}\left(\tan^{-1}x\right)^2\Big|_{-2}^2=\frac{1}{2}\left(\tan^{-1}2\right)^2-\frac{1}{2}\left(\tan^{-1}(-2)\right)^2=0$$

$$D=\int_0^1\left(\sum_{n=1}^{37}(n+1)x^n\right)dx=\sum_{n=1}^{37}x^{n+1}\Big|_0^1=\sum_{n=1}^{37}1-\sum_{n=1}^{37}0=37-0=37$$

$$\frac{Be^{2A}}{C+D}=-\frac{111}{4}\cdot\frac{e^{2\cdot\frac{1}{2}\ln5}}{0+37}=-\frac{111}{4}\cdot\frac{5}{37}=-\frac{15}{4}$$

8. $f(x)=\displaystyle\int_2^x\left(t^4+3t^3-2t^2+17t-14\right)dt$

$$f(2)=0$$

$$f'(x)=x^4+3x^3-2x^2+17x-14\Rightarrow f'(2)=52$$

$$f''(x)=4x^3+9x^2-4x+17\Rightarrow f''(2)=77$$

$f'''(x)=12x^2+18x-4 \Rightarrow f'''(2)=80$

$f^{(4)}(x)=24x+18 \Rightarrow f^{(4)}(2)=66$

$f^{(5)}(x)=24$

$f^{(n)}(x)=0$ for $n \geq 6$

$0+52+77+80+66+24+0+...=299$

9. $$\left.\frac{dy}{dx}\right|_{t=\frac{\pi}{2}}=\left.\frac{dy/dt}{dx/dt}\right|_{t=\frac{\pi}{2}}=\left.\frac{2t+\cos t}{-1-\sin t}\right|_{t=\frac{\pi}{2}}=\frac{2\cdot\frac{\pi}{2}+\cos\frac{\pi}{2}}{-1-\sin\frac{\pi}{2}}=\frac{\pi+0}{-1-1}=-\frac{\pi}{2}$$

$$\left.\frac{d^2y}{dx^2}\right|_{t=\frac{\pi}{2}}=\left.\frac{d\left(dy/dx\right)}{dx}\right|_{t=\frac{\pi}{2}}=\left.\frac{d\left(dy/dx\right)/dt}{dx/dt}\right|_{t=\frac{\pi}{2}}=\left.\frac{\frac{(-1-\sin t)(2-\sin t)-(2t+\cos t)(-\cos t)}{(-1-\sin t)^2}}{-1-\sin t}\right|_{t=\frac{\pi}{2}}$$

$$=\left.\frac{-1-\sin t+2t\cos t}{(-1-\sin t)^3}\right|_{t=\frac{\pi}{2}}=\frac{-1-1+0}{(-1-1)^3}=\frac{-2}{-8}=\frac{1}{4}$$

$$\left.\frac{d^3y}{dx^3}\right|_{t=\frac{\pi}{2}}=\left.\frac{d\left(d^2y/dx^2\right)}{dx}\right|_{t=\frac{\pi}{2}}=\left.\frac{d\left(d^2y/dx^2\right)/dt}{dx/dt}\right|_{t=\frac{\pi}{2}}$$

$$=\left.\frac{\frac{(-1-\sin t)^3(\cos t-2t\sin t)-(-1-\sin t+2t\cos t)3(-1-\sin t)^2(-\cos t)}{(-1-\sin t)^6}}{-1-\sin t}\right|_{t=\frac{\pi}{2}}$$

$$=\left.\frac{(-1-\sin t)(\cos t-2t\sin t)+3\cos t(-1-\sin t+2t\cos t)}{(-1-\sin t)^5}\right|_{t=\frac{\pi}{2}}=\frac{(-2)(-\pi)+0}{(-2)^5}=-\frac{\pi}{16}$$

$$-\frac{\pi}{2}+\frac{1}{4}-\frac{\pi}{16}=\frac{-8\pi+4-\pi}{16}=\frac{4-9\pi}{16}$$

10. The two graphs intersect at the points $(2,1)$ and $(6,9)$.

$$A=\int_2^6\left(2x-3-\frac{1}{4}x^2\right)dx=\left.\left(x^2-3x-\frac{1}{12}x^3\right)\right|_2^6=(36-18-18)-\left(4-6-\frac{2}{3}\right)=\frac{8}{3}$$

$$B=\pi\int_2^6\left((2x-3)^2-\left(\frac{1}{4}x^2\right)^2\right)dx=\pi\int_2^6\left(4x^2-12x+9-\frac{1}{16}x^4\right)dx=\pi\left(\frac{4}{3}x^3-6x^2+9x-\frac{1}{80}x^5\right)\Bigg|_2^6$$

$$=\pi\left(\left(288-216+54-\frac{486}{5}\right)-\left(\frac{32}{3}-24+18-\frac{2}{5}\right)\right)=\frac{368\pi}{15}$$

$x^2=4y$ and $x=\dfrac{y+3}{2}$

$$C=\pi\int_1^9\left((4y)-\left(\frac{y+3}{2}\right)^2\right)dy=\pi\int_1^9\left(\frac{5}{2}y-\frac{1}{4}y^2-\frac{9}{4}\right)dy=\pi\left(\frac{5}{4}y^2-\frac{1}{12}y^3-\frac{9}{4}y\right)\Bigg|_1^9$$

$$=\pi\left(\left(\frac{405}{4}-\frac{243}{4}-\frac{81}{4}\right)-\left(\frac{5}{4}-\frac{1}{12}-\frac{9}{4}\right)\right)=\frac{64\pi}{3}$$

$$\frac{AB}{C}=\frac{\frac{8}{3}\cdot\frac{368\pi}{15}}{\frac{64\pi}{3}}=\frac{46}{15}$$

11. $A=1+0.4(2\cdot 0-1)=0.6$

$1+0.2(2\cdot 0-1)=0.8$

$B=0.8+0.2(2\cdot 0.2-0.8)=0.72$

$1+0.1(2\cdot 0-1)=0.9$

$0.9+0.1(2\cdot 0.1-0.9)=0.83$

$0.83+0.1(2\cdot 0.2-0.83)=0.787$

$C=0.787+0.1(2\cdot 0.3-0.787)=0.7683$

$A+B+C=0.6+0.72+0.7683=2.0883$

12. A: $\dfrac{n}{n^3+1}\le\dfrac{n}{n^3}=\dfrac{1}{n^2}$, which converges by p-series test, so the series converges by comparison

B: $\dfrac{\cos(n\pi)}{n+1}=\dfrac{(-1)^n}{n+1}$; $\dfrac{1}{n+1}\to 0$ and $\dfrac{1}{n+1}$ is decreasing, so the series converges by Alternating Series Test

C: $\left|\dfrac{(-3)^{n+1}}{(2n+3)!!}\cdot\dfrac{(2n+1)!!}{(-3)^n}\right|=\dfrac{3}{2n+3}\to 0<1$, so the series converges by Ratio Test

D: series is geometric with $r=\dfrac{\pi}{4}$, so the series converges

E: $(-1)^n e^{-n^2}\Rightarrow e^{-n^2}\to 0$ and e^{-n^2} is decreasing, so the series converges by Alternating Series Test

F: for $n>0$, $\ln n<n$ (compare the graphs of $y=x$ and $y=\ln x$), so $\frac{1}{\ln n}>\frac{1}{n}$, which diverges by p-series test, so the series diverges by comparison

The series that converge, therefore, are A, B, C, D, and E.

13. $f'(x)=\frac{x\cdot\frac{1}{x}-\ln x\cdot 1}{x^2}=\frac{1-\ln x}{x^2}$ and $\int_1^x \frac{\ln t}{t}\,dt=\frac{1}{2}(\ln t)^2\Big|_1^x=\frac{1}{2}(\ln x)^2-\frac{1}{2}(\ln 1)^2=\frac{1}{2}(\ln x)^2$

$A=f'(1)=\frac{1-\ln 1}{1^2}=1$

$B=\int_1^e f(t)\,dt=\frac{1}{2}(\ln e)^2=\frac{1}{2}$

$C=f'(e^2)=\frac{1-\ln e^2}{(e^2)^2}=-\frac{1}{e^4}$

$D=\int_1^{e^4} f(t)\,dt=\frac{1}{2}(\ln e^4)^2=8$

$\frac{ABD}{C}=\frac{1\cdot 1/2\cdot 8}{-1/e^4}=-4e^4$

14. $f'(x)=f'''(x)=-e^{1-x}$ and $f''(x)=f^{(4)}(x)=e^{1-x}$

Therefore, $A=C=-e^{-2010}$ and $B=D=e^{-2010}$

$$\frac{A}{B}+\frac{A}{C}+\frac{A}{D}+\frac{B}{A}+\frac{B}{C}+\frac{B}{D}+\frac{C}{A}+\frac{C}{B}+\frac{C}{D}+\frac{D}{A}+\frac{D}{B}+\frac{D}{C}=-1+1-1-1-1+1+1-1-1-1+1-1=-4$$