

Answers:

1. $\frac{137}{143}$

2. 17

3. 17

4. $\frac{25}{216}$

5. 23

6. 5

7. $\frac{10}{3}$

8. $\frac{3}{10}$ or 0.3

9. $\frac{41}{63}$

10. 97.35%

11. mean, range, IQR

12. $\frac{49}{4}$ or 12.25

13. mean

14. $\frac{125}{1296}$

15. $\frac{3}{8}$

16. $\frac{2}{3}$

17. -2

18. no

19. $\frac{274}{5525}$

20. 1.65

21. Type II error

22. $\frac{3}{5}$

23. D

24. 7

25. $\frac{1}{40}$

Solutions:

$$1. \quad 1 - P(\text{all girls}) = 1 - \frac{\binom{9}{5}}{\binom{15}{5}} = 1 - \frac{126}{3003} = \frac{2877}{3003} = \frac{137}{143}$$

$$2. \quad \text{IQR} = Q3 - Q1 = 42 - 25 = 17$$

3. 80th percentile is 16th number out of 20, which is 17

4. Dice could be 1,5,6 (6 combinations); 2,4,6 (6 combinations); 2,5,5 (3 combinations); 3,3,6 (3 combinations); 3,4,5 (6 combinations); and 4,4,4 (1 combination); for a total of 25 combinations. There are a total of $6^3 = 216$ different rolls, so the probability is $\frac{25}{216}$

$$5. \quad 10(0.3) + 20(0.3) + 30(0.2) + 40(0.2) = 3 + 6 + 6 + 8 = 23$$

$$6. \quad \frac{1}{6} = \frac{\binom{x}{2}}{\binom{9}{2}} = \frac{x(x-1)}{9 \cdot 8} \Rightarrow x(x-1) = 12 \Rightarrow x = 4, \text{ so there are 5 black marbles}$$

$$7. \quad 2.5 \cdot \frac{20}{\sqrt{225}} = \frac{50}{15} = \frac{10}{3}$$

$$8. \quad P(B) = 3P(A), P(C) = 2P(B), P(A) + P(B) + P(C) = 1$$

$$P(A) + 3P(A) + 6P(A) = 1 \Rightarrow 10P(A) = 1 \Rightarrow P(A) = \frac{1}{10}$$

$$\text{Therefore, } P(B) = \frac{3}{10}.$$

$$9. \quad \frac{\binom{6}{1} + \binom{6}{2} + \binom{6}{3}}{2^6 - 1} = \frac{6 + 15 + 20}{63} = \frac{41}{63}$$

10. From -2 to 0 s.d.s is 47.5%, and from 0 to 3 s.d.s is 49.85%, so the total is $47.5\% + 49.85\% = 97.35\%$

11. five-number summary is minimum, Q1, median, Q3, maximum, so the ones that are not included are mean, range, and IQR

$$12. \quad 1 \cdot \frac{1}{36} + 2 \cdot \frac{2}{36} + 3 \cdot \frac{2}{36} + 4 \cdot \frac{3}{36} + 5 \cdot \frac{2}{36} + 6 \cdot \frac{4}{36} + 8 \cdot \frac{2}{36} + 9 \cdot \frac{1}{36} + 10 \cdot \frac{2}{36} + 12 \cdot \frac{4}{36} + 15 \cdot \frac{2}{36} \\ + 16 \cdot \frac{1}{36} + 18 \cdot \frac{2}{36} + 20 \cdot \frac{2}{36} + 24 \cdot \frac{2}{36} + 25 \cdot \frac{1}{36} + 30 \cdot \frac{2}{36} + 36 \cdot \frac{1}{36} = \frac{441}{36} = \frac{49}{4}$$

13. The mean is greater because it is less resistant and will be pulled to the right by the data.

$$14. \quad \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} = \frac{125}{1296}$$

15. $P(2 \text{ boys, 2 girls in middle}) = \frac{\binom{4}{2}}{2^4} = \frac{6}{16} = \frac{3}{8}$
16. Winning sequences for the first person would be T or HHT or HHHHT, etc. (an even number of heads followed by a tails). The probabilities are $(\frac{1}{2})^1, (\frac{1}{2})^3, (\frac{1}{2})^5, \dots$, so the probability would be $\frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \dots = \frac{\frac{1}{2}}{1 - \frac{1}{4}} = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$
17. residual = observed – expected = 95 – 97 = –2
18. no, don't reject because the p -value must be compared to $\alpha/6 = .00833$, so $p > \alpha/6$
19. First, choose the suit (4), then choose the number of total flushes, which is $\binom{13}{3}$, then subtract the number of straights, which is 4 suits times 12 possible beginning cards (all cards except King). Therefore, the probability is:
- $$\frac{\binom{4}{1} \cdot \binom{13}{3} - \binom{4}{1} \cdot 12}{\binom{52}{3}} = \frac{4 \cdot 286 - 4 \cdot 12}{22100} = \frac{274}{5525}$$
20. The middle 90% is between $z = -1.645$ and $z = 1.645$, so to the nearest hundredth, the absolute value is 1.65.
21. You rejected the null hypothesis when the alternative hypothesis was true. This is a correct decision.
22. $P(\text{lasts 150 hours} | \text{lasts 100 hours}) = \frac{P(\text{lasts 150 hours} \cap \text{lasts 100 hours})}{P(\text{lasts 100 hours})}$
- $$= \frac{P(\text{lasts 150 hours})}{P(\text{lasts 100 hours})} = \frac{\frac{1}{5}}{\frac{1}{3}} = \frac{3}{5}$$
23. If all data are the same, A is true. If data consist of 3 equally spaced numbers and the middle one is small with the others spread out a lot, B could be true. If all the bottom fourth of the data are equal and the top fourth of the data are equal, C could be true. Therefore, all are possible (D).
24. $P(X \geq 2) \geq 0.5 \Rightarrow 1 - P(X = 0) - P(X = 1) \geq 0.5 \Rightarrow P(X = 0) + P(X = 1) \leq 0.5$
- $$\left(\frac{3}{4}\right)^n + \binom{n}{1} \left(\frac{1}{4}\right) \left(\frac{3}{4}\right)^{n-1} \leq 0.5 \Rightarrow \left(\frac{3}{4}\right)^{n-1} \left(\frac{3+n}{4}\right) \leq 0.5.$$
- Inspection shows the first value of n that works is $n = 7$.
25. 74 is 2 standard deviations above the mean, so that region constitutes 2.5% of the total area. As a fraction, this is $\frac{1}{40}$.