Answers:

1. 1
2. $0, \pm\sqrt{2}$
3. 2
4. $-\frac{\pi}{1+\pi^2}$
5. $3s^2$
6. 1, 3
7. $15/4$
8. $1/2\pi$
9. 27
10. 6
11. 7
12. $y' = \frac{10x}{x^4 - 25}$
13. $y' = x^{x^2}\left(x + 2x\ln x\right)$
14. $y + \sqrt{2} = \frac{\sqrt{2}}{3}\left(x-1\right)$
15. $-1/3$
16. $-\left(1+x^2\right)^{-3/2}$
17. $\ln 2$
18. $-\frac{1}{2}\cos\left(x^2 + 4x - 6\right) + c$
19. $y = \frac{1+x}{1-x}$
20. $\pi\left(\frac{e-1}{e}\right)$
21. -4
22. 6π
23. 8
24. -1
25. $-1/4$

Solutions:

1. $c^2=\dfrac{\left(\sqrt{3}\right)^3-0^3}{3\left(\sqrt{3}-0\right)}\Rightarrow c^2=1\Rightarrow c=1$ because c is in the interval $\left[0,\sqrt{3}\right]$

2. $\left(\dfrac{1}{4}t^4-\dfrac{1}{2}t^2\right)\Bigg|_0^x=\dfrac{1}{4}x^4-\dfrac{1}{2}x^2$ and $\dfrac{1}{3}\left(\dfrac{1}{2}t^2-\dfrac{1}{4}t^4\right)\Bigg|_{\sqrt{2}}^x=\dfrac{1}{3}\left(\dfrac{1}{2}x^2-\dfrac{1}{4}x^4\right)$, so

$0=\dfrac{4}{3}\left(\dfrac{1}{4}x^4-\dfrac{1}{2}x^2\right)=\dfrac{1}{3}x^2\left(x^2-2\right)$, so $x=0$ or $x=\pm\sqrt{2}$

3. $\lim\limits_{x\to 0}\dfrac{\sin 5x-\sin 3x}{x}=\lim\limits_{x\to 0}\dfrac{5\cos 5x-3\cos 3x}{1}=5-3=2$

4. $f'(x)=\dfrac{\left(1+x^2\right)\left(x\cos x+\sin x\right)-\left(x\sin x\right)\left(2x\right)}{\left(1+x^2\right)^2}\Rightarrow f'(\pi)=\dfrac{\left(1+\pi^2\right)\left(-\pi\right)-0}{\left(1+\pi^2\right)^2}=-\dfrac{\pi}{1+\pi^2}$

5. $V=s^3\Rightarrow\dfrac{dV}{ds}=3s^2$

6. $f'(x)=x^2-4x+3=(x-1)(x-3)$, so $f'(x)=0$ when $x=1$ or $x=3$

7. Using the first and last equation yields $f(0)=\frac{1}{2}$ and $g(0)=4$. This implies that

$f'(0)=16$ and $g'(0)=8$. Therefore, $h'(0)=\dfrac{g(0)f'(0)-f(0)g'(0)}{g^2(0)}=\dfrac{4\cdot 16-\frac{1}{2}\cdot 8}{4^2}$

$=\dfrac{60}{16}=\dfrac{15}{4}$

8. $\dfrac{dV}{dt}=4\pi r^2\dfrac{dr}{dt}\Rightarrow 50=4\pi(5)^2\dfrac{dr}{dt}\Rightarrow\dfrac{dr}{dt}=\dfrac{1}{2\pi}$

9. $\lim\limits_{x\to 3}\dfrac{x^3-27}{x-3}=\lim\limits_{x\to 3}\dfrac{3x^2}{1}=3(3)^2=27$

10. $V=x(36-2x)^2=4x^3-144x^2+1296x\Rightarrow V'=12x^2-288x+1296=12(x-18)(x-6)$,

and for values of $x<6, f'(x)>0$, while for values of x, $6<x<18$, $f'(x)<0$.

Therefore, the box has a maximum value for $x=6$ since this function only makes sense for value of x on the interval $[0,18]$.

11. $s(t)=t^3-t^2+8t \Rightarrow s'(t)=3t^2-2t \Rightarrow s''(t)=6t-2 \Rightarrow s''\left(3/2\right)=6\left(3/2\right)-2=7$

12. $y=\ln\sqrt{\frac{x^2-5}{x^2+5}}=\frac{1}{2}\left(\ln\left(x^2-5\right)-\ln\left(x^2+5\right)\right) \Rightarrow y'=\frac{1}{2}\left(\frac{2x}{x^2-5}-\frac{2x}{x^2+5}\right)=\frac{10x}{x^4-25}$

13. $\ln y=x^2\ln x \Rightarrow \frac{1}{y}\frac{dy}{dx}=x^2\cdot\frac{1}{x}+2x\ln x \Rightarrow \frac{dy}{dx}=y\left(x+2x\ln x\right)=x^{x^2}\left(x+2x\ln x\right)$

14. $6x^2=2y\frac{dy}{dx} \Rightarrow \frac{dy}{dx}=\frac{3x^2}{y}$, so the normal curve has slope $\frac{dy}{dx}=-\frac{y}{3x^2}$, which at the point $\left(1,-\sqrt{2}\right)$ has value $\frac{\sqrt{2}}{3}$. Therefore, the equation of the normal line is $y+\sqrt{2}=\frac{\sqrt{2}}{3}(x-1)$

15. $2x^2y\frac{dy}{dx}+2xy^2+3x^2-2-4y^3\frac{dy}{dx}-6\frac{dy}{dx}=0$, which reduces to $-2-6\frac{dy}{dx}=0$ when $x=y=0$. Solving this gives $\frac{dy}{dx}=-\frac{1}{3}$

16. $f(x)=\frac{\sqrt{1+x^2}-x}{\sqrt{1+x^2}}=1-x\left(1+x^2\right)^{-1/2} \Rightarrow f'(x)=-x\left(-\frac{1}{2}\left(1+x^2\right)^{-3/2}\cdot 2x\right)-\left(1+x\right)^{-1/2}$

$=\frac{x^2}{\left(1+x^2\right)^{3/2}}-\frac{1+x^2}{\left(1+x^2\right)^{3/2}}=-\frac{1}{\left(1+x^2\right)^{3/2}}=-\left(1+x^2\right)^{-3/2}$

17. $\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\ldots+\frac{1}{n+n}\right)=\lim_{n\to\infty}\left(\sum_{i=1}^{n}\frac{1}{1+\frac{i}{n}}\right)\frac{1}{n}=\int_0^1\frac{1}{1+x}dx=\ln\left|1+x\right|\Big|_0^1=\ln 2-\ln 1=\ln 2$

18. Making the substitutions $u=x^2+4x-6$ and $du=2\left(x+2\right)dx$, the integral becomes $\frac{1}{2}\int\sin u\,du=-\frac{1}{2}\cos u+c=-\frac{1}{2}\cos\left(x^2+4x-6\right)+c$

19. $\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2} \Rightarrow \tan^{-1} y = \tan^{-1} x + c \Rightarrow y = \frac{x+\tan c}{1-x\tan c} \Rightarrow 1 = \frac{0+\tan c}{1-0\tan c} = \tan c$, so the function is $y = \frac{1+x}{1-x}$.

20. $2\pi\int_0^1 xe^{-x^2}dx = 2\pi\left(-\frac{1}{2}e^{-x^2}\right)\Big|_0^1 = \pi\left(-e^{-1}+1\right) = \pi\left(\frac{e-1}{e}\right)$

21. $\left.\frac{d^2y}{dx^2}\right|_{t=\frac{\pi}{3}} = \left.\frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}\right|_{t=\frac{\pi}{3}} = \left.\frac{\frac{d}{dt}\left(\frac{\sin t}{1-\cos t}\right)}{1-\cos t}\right|_{t=\frac{\pi}{3}} = \left.\frac{\frac{(1-\cos t)\cos t - \sin^2 t}{(1-\cos t)^2}}{1-\cos t}\right|_{t=\frac{\pi}{3}} = \frac{-2}{1/2} = -4$

22. $2\cdot\frac{1}{2}\cdot 4\int_0^\pi (1+\cos\theta)^2\,d\theta = 4\int_0^\pi\left(1+2\cos\theta+\cos^2\theta\right)d\theta = 4\int_0^\pi\left(\frac{3}{2}+2\cos\theta+\frac{1}{2}\cos 2\theta\right)d\theta$
$= \left(6\theta+8\sin\theta+\sin 2\theta\right)\Big|_0^\pi = 6\pi$

23. $2\int_0^\pi \sqrt{(1-\cos\theta)^2+\sin^2\theta}\,d\theta = 2\int_0^\pi\sqrt{2-2\cos\theta}\,d\theta = 4\int_0^\pi \sin\left(\theta/2\right)d\theta = -8\cos\left(\theta/2\right)\Big|_0^\pi = 8$

24. $\lim_{t\to-\infty}\int_t^0 xe^x dx = \lim_{t\to-\infty}\left(xe^x - e^x\right)\Big|_t^0 = \lim_{t\to-\infty}\left(-1-te^t+e^t\right) = -1+\lim_{t\to-\infty}\left(\frac{t}{e^{-t}}\right) = -1+\lim_{t\to-\infty}\left(\frac{1}{-e^{-t}}\right)$
$= -1$

25. Let $f(x) = \ln(1+x)$. The coefficient is $\frac{f^{(4)}(0)}{4!} = \frac{-\frac{6}{(1+0)^4}}{24} = -\frac{1}{4}$.