The acronym NOTA denotes that "None of These Answers" are correct. The acronym DNE stands for "Does Not Exist". The domain and range of functions are assumed to be either the real numbers or an appropriate subset, unless otherwise stated. $[x]$ represents the greatest integer $\leq x$. Good luck and have fun!

1. What is the area between the curve $a(x) = x^2 + x + 1 + \frac{1}{x}$ and the x-axis on the interval (2, 8)?
A. $198 + 2\ln 2$ B. $204 + 2\ln 2$ C. $210 + 2\ln 2$ D. $214 + 2\ln 2$ E. NOTA

2. Find the average value of the function $f(x) = x^2 + x + 1 + \frac{1}{x} + ...$ on the interval (2, 4).
A. $\frac{40}{3} + \ln\sqrt{2}$ B. $\frac{80}{3} + \ln 2$ C. $\frac{40}{3} + \ln\sqrt{3}$ D. $\frac{80}{3} + \ln 3$ E. NOTA

3. An annulus (a "small circle inside a larger circle") currently has inner radius 5cm and outer radius 13cm. If the inner radius increases at 2 cm/sec, what is the rate of change of the area of the annulus(in cm^2/sec) when the inner radius equals the outer radius?
A. -52π B. -26π C. 26π D. 52π E. NOTA

4. What is the area bound by the curve $y = \dfrac{\cos(x) - \sin(x)}{1 + \sin(2x)}$ and the x-axis on the interval $\left[0, \frac{\pi}{6}\right]$?
A. $\sqrt{3} - 2$ B. $2 - \sqrt{3}$ C. $\sqrt{3} - 1$ D. $2 + \sqrt{3}$ E. NOTA

5. Find the area bound in the Argand plane by the curve $|2a - ib| - 9 = 0$.
A. $\frac{9\pi}{8}$ B. $\frac{9\pi}{4}$ C. $\frac{9\pi}{2}$ D. 9π E. NOTA

6. Let $f(x) = x^4 + 6x^2 + 1$ and let $g(x) = 4x^3 + 4x + 16$. Find the area of the region bound above by $g(x)$ and below by $f(x)$.
A. $\frac{64}{5}$ B. $\frac{128}{5}$ C. $\frac{256}{5}$ D. $\frac{512}{5}$ E. NOTA

7. The region bound above $f(x) = e^{-x^2}$, below by the y-axis, and to the left by $x = 0$ (unbounded to the right) is rotated about the y axis. What is the volume of the solid formed? (It's finite!)
A. $\frac{\pi}{4}$ B. $\frac{\pi}{2}$ C. π D. 2π E. NOTA

8. What is the area bound by $y = \arcsin(x) + \arccos(x)$, $x = 0, x = \frac{1}{2}$, and $y = 0$?
A. $\frac{\pi}{12} + 2 - \sqrt{3}$ B. $\frac{\pi}{4}$ C. $\frac{\pi}{6} + 2 - \sqrt{3}$ D. $\frac{\pi}{2}$ E. NOTA

For questions 9-11, consider the region R bound by $x^2 + y^2 - 4x = 2y + 20$.

9. What is the volume of the solid formed when R is rotated about the line $3x - 4y = 2$?
A. $\frac{250\pi}{3}$ B. $\frac{500\pi}{3}$ C. $46\pi^2$ D. $92\pi^2$ E. NOTA

10. The lines $y = 1$ and $y = 3.5$ cut R into 3 regions. Find the area of R between $y = 1$ and $y = 3.5$.
A. $\frac{25\pi}{12} - \frac{25\sqrt{3}}{8}$ B. $\frac{25\pi}{12} + \frac{25\sqrt{3}}{8}$ C. $\frac{25\pi}{6} - \frac{25\sqrt{3}}{4}$ D. $\frac{25\pi}{6} + \frac{25\sqrt{3}}{4}$ E. NOTA

11. Let A be the part of R above the line $y = 2$. Which integral below gives the same numerical value as the volume of the solid formed when A is rotated about $y = 2$?
A. $2\pi \int_1^5 (5-x)\sqrt{25-x^2}dx$ B. $4\pi \int_1^5 (5-x)\sqrt{25-x^2}dx$ C. $\pi \int_{2-2\sqrt{6}}^{2+2\sqrt{6}} \left((\sqrt{25-x^2}+1)^2 - 4\right) dx$
D. $2\pi \int_{2-2\sqrt{6}}^{2+2\sqrt{6}} \left((\sqrt{25-x^2}+1)^2 - 4\right) dx$ E. NOTA

12. A rectangle has vertices at $(k, 0)$, $(-k, 0)$, and two vertices on the part of the parabola $y = 4 - x^2$ where $y > 0$. Find the positive value of k that maximizes the area of the rectangle.
A. $\frac{\sqrt{3}}{3}$ B. $\frac{\sqrt{3}}{2}$ C. $\frac{2\sqrt{3}}{3}$ D. $\sqrt{3}$ E. NOTA

13. Let $f(x) = -x^2 + 2x$, initially. Hurricane Zeta begins to push $f(x)$ upward such that it is moving in the positive y-direction at 2 units/sec. At what rate, in units^2/sec, is the area bound by $f(x)$ and the x-axis changing four seconds after Hurricane Zeta starts?
A. 6 B. 8 C. 12 D. 16 E. NOTA

14. Consider the triangle with vertices at (1, 0), (2, 2), and (6, 1). I pick a vertex at random and rotate this triangle about the vertex. What's the largest possible volume of the resulting solid?
A. $9\sqrt{2}\pi$ B. $9\sqrt{5}\pi$ C. 27π D. 54π E. NOTA

15. The density of a gas is not constant. Suppose that I have 10π grams of xenon, a noble gas, which I store in a container in the shape of the figure formed when the region bound by $y = \sqrt{k - x^2}$, $x = 0$, and $y = 0$ is revolved about the $y-$axis (and has volume, in cm^3, which corresponds to the value of k). If the value of k is currently 3cm, and is changing at a constant 2 cm/sec, at what rate is the density of this xenon changing (in g/cm^3) in 3 seconds?
A. $-\frac{5}{1458}$ B. $-\frac{5}{729}$ C. $-\frac{5}{54}$ D. $-\frac{5}{27}$ E. NOTA

16. Consider the region bound by $f(x) = x^3$, the x-axis, $x = 0$, and $x = 4$. If the line $x = k$ divides this region into two pieces of equal area, what is k^2?
A. 4 B. $4\sqrt{2}$ C. 8 D. $8\sqrt{2}$ E. NOTA

17. Suppose f is a continuous function and $\int_{-2}^{2} f(x)dx = 0$ and $\int_{0}^{2} f(x)dx = 2$. What is the area bound by $f(x)$, the x-axis, and the lines $x = -2$ and $x = 2$?

A. 0 B. 1 C. 2 D. 4 E. NOTA

18. Consider the region bound by the lines $y = 0$, $x = 0$, $x = 2$, and $f(x) = 3^x$. Circular cross-sections to this region perpendicular to the x-axis are drawn. What is the volume of the resulting solid?

A. $\frac{5\pi}{\ln(3)}$ B. $\frac{10\pi}{\ln(3)}$ C. $\frac{20\pi}{\ln(3)}$ D. $\frac{40\pi}{\ln(3)}$ E. NOTA

19. Approximate the area bound by the curve $f(x) = x^2$ and the x-axis from $x = 0$ to $x = 2$ using the right-hand rectangle approximation method with the first subinterval being (0, 1) and each subsequent subinterval with length of one-half of that of the subinterval to its left.

A. $\frac{59}{42}$ B. $\frac{40}{21}$ C. $\frac{59}{21}$ D. $\frac{80}{21}$ E. NOTA

20. Find the area bound by the curve $f(x) = x^2$ and the x-axis from $x = 0$ to $x = 2$.

A. $\frac{4}{3}$ B. 4 C. $\frac{16}{3}$ D. 8 E. NOTA

21. Consider rectangle A with vertices at (0, 0), $(0, m)$, $(n, 0)$, and (m, n). It is known that the perimeter of rectangle A is 24 units. Let Set R be the set of all possible solids formed when rectangle A is rotated about the x-axis. Find the average value of the volumes of the solids in set R.

A. 288π B. 576π C. 1152π D. 1728π E. NOTA

22. Consider the curve $f(x) = \frac{4}{3}x^3$. If the region bound by $f(x)$, $x = 0$, $x = 1$ and the x-axis is rotated about the x-axis to form a figure in the shape of a very wide cup–that is, closed off at the $x = 0$ end and open at the $x = 1$ end–then what is the surface area of the outside of this figure? (i.e. don't double-count)

A. $\frac{17\sqrt{17}-1}{72}\pi$ B. $\frac{17\sqrt{17}+1}{72}\pi$ C. $\frac{17\sqrt{17}-1}{36}\pi$ D. $\frac{17\sqrt{17}+1}{36}\pi$ E. NOTA

23. Find the volume of the solid formed when the region bound by $f(x) = x^4$, $x = 0$, and $y = 16$ is rotated about the y-axis.

A. $\frac{8\pi\sqrt[4]{2}}{5}$ B. $\frac{16\pi\sqrt[4]{2}}{5}$ C. $\frac{4096\pi}{9}$ D. $\frac{8192\pi}{9}$ E. NOTA

24. Find the volume of the solid formed when the region bound by $f(x) = x^4$, $x = 0$, and $y = 16$ is rotated about the x-axis.

A. $\frac{8\pi\sqrt[4]{2}}{5}$ B. $\frac{16\pi\sqrt[4]{2}}{5}$ C. $\frac{4096\pi}{9}$ D. $\frac{8192\pi}{9}$ E. NOTA

25. Suppose that on the interval (0, 10), the function $f(x)$ is decreasing, concave up, and greater than zero. Which of the following methods, with 5 equal subdivisions, overestimates the value of $\int_0^{10} f(x)dx$ regardless of what $f(x)$ is?

I. Left Rectangular
II. Right Rectangular
III. Midpoint
IV. Trapezoid

A. I, III, IV only B. II, IV only C. I, III only D. I, IV only E. NOTA

26. Use the left rectangle approximation method with four equal subdivisions to approximate the area bounded by the curve $y = x^2 + 2$ and the x-axis between $x = 0$ and $x = 4$.

A. 14 B. 22 C. 29 D. 38 E. NOTA

27. Consider the curve $f(x) = ax^2$ where $a > 0$. Let Q be the region bound by $f(x)$, $x = 0$, $y = 0$, and $x = a$. Let A be the volume of the solid formed when Q is rotated about the x-axis, and let R be the volume of the solid formed when Q is rotated about the y-axis. Find the smallest positive integer greater than the value of a for which $A = R$.

A. 1 B. 2 C. 3 D. 4 E. NOTA

28. Find the volume of the solid formed when the region bound by $f(x) = \sin(x)$, $x = 0$, $y = 0$, and $y = 1$ is rotated about the y-axis.

A. $\frac{\pi^3}{8} - \pi$ B. $\frac{\pi^3}{4} - 2\pi$ C. $\frac{\pi^2}{4}$ D. 2π E. NOTA

29. Find the volume of the solid formed when the region bound by $f(x) = \sin(x)$, $x = 0$, $y = 0$, and $y = 1$ is rotated about the x-axis.

A. $\frac{\pi^3}{8} - \pi$ B. $\frac{\pi^3}{4} - 2\pi$ C. $\frac{\pi^2}{4}$ D. 2π E. NOTA

30. Consider triangle WLH with vertices W(1, 3), L(2, 2), and H(a^2, b^2). Given that currently $a = 2$ and $b = 2$, and that a is increasing at 2 units/sec and b at 3 units/sec, what is the absolute value of the rate of change of the area of triangle WLH four seconds from now?

A. 24 B. 29 C. 31 D. 58 E. NOTA