2009 Theta School Bowl

1.

Let $A\sqrt{B}$ equal the area of a triangle whose sides are 10, 12, and 6.

Let $C\pi$ equal the area of a circle inscribed in an equilateral triangle with side length 6.

Let *D* be the least integer in the domain of $y = \sqrt{2x^3 + 11x^2 - 7x - 6}$.

Let *E* be the value of $\left[\log_3 \frac{24}{11}\right]$, where [ ] represents the greatest integer function.

Use $\log_3 8 \approx 1.9$ and $\log_3 11 \approx 2.2$.

Find the sum of *A*, *B*, *C*, *D*, and *E*.

2.

Let *A* be the radius of a circle inscribed in a 60 degree sector of a circle whose diameter is 10.

The number of volunteers varies directly as the square of the number of commodores and inversely as twice the number of blue raiders. When there are 8 commodores and 5 blue raiders, there are 768 volunteers. Find *B*, the number of volunteers, when there are 7 commodores and 7 blue raiders.

Let *C* be the probability of drawing a queen or a red jack from a standard deck of playing cards that is missing all black odd-numbered cards less than 5. (Aces are not considered odd-numbered!)

Find the product of *A*, *B*, and *C*.

3.

Let *A* represent the length of the altitude to the hypotenuse of the right triangle whose legs are 5 and 12.

The sides of a triangle are 5, 3, 4. Let *B* represent the radius of its inscribed circle.

Let *C* be the determinant of the matrix $\begin{bmatrix} 10 & 1 & 5 \\ 3 & 0 & 12 \\ 2 & -7 & -8 \end{bmatrix}$, when each entry is multiplied by 2.

Let *D* be the distance between the foci on the graph of $3x^2 + y^2 + 12x + 6y - 6 = 0$.

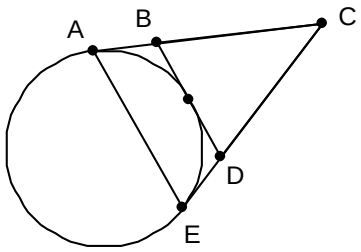
What is the most commonly used digit in your four answers?

4.
Find the positive difference between the minimum and maximum values of the objective function $z = 5x + 7y$ where $x \geq 0$ and $y \geq 0$ subject to the following constraints:

$$\left.\begin{array}{l} 2x + 3y \geq 6 \\ 3x - y \leq 15 \\ -x + y \leq 4 \\ 2x + 5y \leq 27 \end{array}\right\}.$$

5.
A = the area of a semicircle in which a triangle with sides 18, 80 and 82 is inscribed.

X represents the perimeter of triangle ACE given that *AB* = 3, *DC* = 4, and the radius of the circle is 5. $\overline{AC}$, $\overline{EC}$, and $\overline{BD}$ are tangent to the circle, and $\overline{BD} \| \overline{AE}$.



Let *F* be the sum of the integer solutions to $\dfrac{x^2 + 2x - 8}{4x^3 - 32} > 0$ which are less than 5.

Let *G* be the number of distinct arrangements of THETASCHOOLBOWL if the letters in BASKETCASE are not used.

Find twice the sum of *A*, *X*, *F*, and *G*.

6.
A = length of apothem in a regular hexagon whose sides are 4.

B= the sum of all integers *x* such that $|3x - 12| + 2 \leq |x + 5| - 3$.

C= the value of $\log xy$ if $\log xy^3 = 4$ and $\log x^2 y = 9$.

Find $10(A + B + C)$.

7.

$\sqrt{A}$ = length of space diagonal in a rectangular solid whose dimensions are 6, 4, and 9.

B = length of a post perpendicular to the floor and touching the ceiling of a semicircular tunnel whose diameter is 10. If measured along a diameter, the bottom of the post is 2 feet from the side of the tunnel.

C = find the sum of the elements in the inverse of $\begin{bmatrix} 3 & \frac{1}{7} \\ -\frac{1}{2} & \frac{3}{14} \end{bmatrix}$

The Brady family has 6 kids: 3 boys and 3 girls. Let *D* = the number of ways the 6 kids can be seated in a row of 6 chairs, if all 3 boys and all 3 girls are not seated together.

Find the sum of the prime answers for *A*, *B*, *C*, and *D*.

8.

A = the number of common tangents for two externally tangent circles.

B = the *x*-value such that $\frac{x}{x-2}+\frac{1}{x-4}=\frac{2}{x^2-6x+8}$

C = the sum of the roots taken 2 at a time for the equation $4x^8-7x^7-6x^6+8x^4-3x^2+2x+7=0$.

Find C^{AB}.

9.

A = distance from (5, 3) to $y=\frac{2}{3}x-1$, written in the form $\frac{T\sqrt{U}}{V}$

B = area of a triangle in which two consecutive sides are 8 & 10 and form an angle of 135° in the triangle, written in the form $W\sqrt{X}$.

C = the value for *x* such that $\log_{0.2}(x-1)+\log_{25}9=-2$.

D = the highest power of 9 which is a factor of 50!

Simplify $\sqrt{(T)(U)(V)(W)(X)(C)(D)}$.

10.
A = tan (60°)

B = the value of x which satisfies $\begin{vmatrix} x & 3 & 2 \\ 1 & 0 & -5 \\ 10 & -2 & -3 \end{vmatrix} = 17$.

$y = \dfrac{4x^2 - 8x + 2}{6x^2 - 22x + 20}$ has vertical asymptotes at x = S and x = T and horizontal asymptote at y = U. Let C = STU.

Find ABC.

11.
A = the measure of one interior angle of a regular pentadecagon.

B = the number of T's in the final column of the truth table for $(\neg p \vee q) \rightarrow (p \wedge \neg q)$

$C = \left[(\log_2 x)(\log_3 y)(\log_x 8)(\log_y 81)\right]^2$

$D = P(13)$ when $P(x) = x^3 - 12x^2 - 26x + 169$

Find the sum of the even values of A, B, C, D.

12.
An ant is crawling on a ball whose diameter is 8. The ant travels a distance that is a 90° rotation in respect to the center of the ball. Let A = distance the ant traveled.

B = the sum of all x such that the area of triangle ABC with vertices (x,7), (–5, x), and (x, 1) is 10 square units.

C = the area of the quadrilateral formed by connecting the endpoints of the major and minor axes of the ellipse defined by $x^2 + 6x + 25y^2 + 100y + 9 = 0$.

Find the lateral surface area of a cone with base circumference A and slant height (B + C).