

1) Let  $F(x) = e^{x^2}$  for all  $x \geq 0$ . Find  $F^{-1}(x)$

- A)  $\ln x$                       B)  $\ln \sqrt{x}$                       C)  $-\sqrt{\ln x}$                       D)  $\frac{1}{\sqrt{\ln x}}$                       E) NOTA

2) Find the Taylor series expansion of  $e^{-2x^2}$  about  $x=0$ .

- A)  $\sum_{n=0}^{\infty} \frac{(-1)^n (x)^{2n}}{n!}$                       B)  $\sum_{n=0}^{\infty} \frac{(-1)^n (2x)^n}{n!}$                       E) NOTA  
 C)  $\sum_{n=0}^{\infty} \frac{(-1)^n (2x)^{2n}}{n!}$                       D)  $\sum_{n=0}^{\infty} \frac{(2x)^{2n}}{n!}$

3) Who is actually credited with the discovery of the constant  $e$ ?

- A) Bernoulli                      B) Euclid                      C) Euler                      D) Napier                      E) NOTA

4) Simplify  $e^{2 \ln(5x) - \ln(25) + \ln(2x)}$

- A)  $\frac{72x^3}{25}$                       B)  $72x^2$                       C)  $\frac{12x^3}{25}$                       D)  $\ln \frac{72x^3}{25}$                       E) NOTA

5)  $\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x}\right)^{3x} =$

- A)  $e^{-6}$                       B)  $e^2$                       C)  $e^i$                       D)  $e^6$                       E) NOTA

6) Find the sum of all values of  $n$  such that  $y = e^{nx}$  is a solution of the following differential equation:

$$\frac{d^2y}{dx^2} - 6 \frac{dy}{dx} + 5y = 0$$

- A) -2                      B) 6                      C) 2                      D) 4                      E) NOTA

7) Reduce  $\frac{\ln |\cos(120^\circ)|^4}{\ln(\sin 30^\circ)}$

- A)  $\ln 8$                       B) 4                      C)  $\ln 4$                       D)  $\ln 2$                       E) NOTA

8) If one uses Euler's Formula for Polyhedrons, then one knows that  $F + V - E = ?$

- A) 2                      B) 1                      C) 4                      D) 7                      E) NOTA

9) If  $f(x) = 3 - 4e^{x+7}$  find  $f^{-1}(x)$  with  $x < 3$

- A)  $\ln \frac{3-x}{3+x} - 7$       B)  $-4e^{x+7}$       C)  $\ln \frac{3-x}{4} - 7$       D)  $\ln 3 - x$       E) NOTA

10) Solve for x:  $e^x - 12e^{-x} - 1 = 0$

- A)  $\ln 4, \ln -3$       B)  $\ln(\sqrt{37} + 6)$       C)  $2\ln 2$       D)  $\ln 2$       E) NOTA

11) If  $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$ , then which of the following answers represents  $\lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^x$ ?

- A)  $\frac{1}{e}$       B)  $-e$       C)  $e-1$       D)  $e^2$       E) NOTA

12) What is the 19<sup>th</sup> digit of  $e$ ?

- A) 5      B) 3      C) 6      D) 2      E) NOTA

13)  $\int x^2 \ln(x) dx$

- A)  $x - 2x \ln x + c$       B)  $x^3 \left(\frac{\ln(x)}{3} - \frac{1}{9}\right) + c$   
 C)  $x^2 \left(\frac{\ln(x)}{3} - \frac{1}{3}\right)$       D)  $x^3 \left(\frac{\ln(x)}{3}\right) + c$       E) NOTA

14) A pot of water is brought to a boil in the beautiful California outdoors with an ambient temperature of  $20^\circ\text{C}$ . After 15 minutes the temperature of the water has decreased from boiling to  $75^\circ\text{C}$ . Find the temperature after another 10 minutes.

- A)  $20 + 75 e^{\ln\left(\frac{15}{10}\right)}^\circ\text{C}$       B)  $100e^{-\frac{1}{3}}^\circ\text{C}$       E) NOTA  
 C)  $63.5^\circ\text{C}$       D)  $20 + 80e^{\left(\frac{10}{15}\right) \cdot \ln\left(\frac{15}{10}\right)}^\circ\text{C}$

15) Solve  $\ln(2x - 1) + \ln(3x + 1) = 1$  for  $x$ .

- A)  $\frac{1 + \sqrt{-23 + 24e}}{12}$       B)  $\frac{1 + \sqrt{23 + 24e}}{12}$       C)  $\frac{1 + \sqrt{23 + 24e}}{12}$       D)  $\frac{1}{2} + \frac{-1}{3}$       E) NOTA

16)  $\lim_{x \rightarrow 0} (1 - x)^{\frac{1}{x}}$

- A)  $\frac{1}{e}$       B)  $e$       C)  $\frac{1}{2e}$       D)  $x - 1$       E) NOTA

17)  $e$  is the unique number with the property that the area of the region bounded by the  $x$ -axis, the vertical lines  $x = 1$  and  $x = e$ , and the function \_\_\_\_\_ is equal to 1.

- A)  $y = \frac{e}{x}$       B)  $y = \frac{1}{x}$       C)  $y = e^x$       D)  $y = \frac{1}{e}$       E) NOTA

18) In 1792, Carl Friedrich Gauss devised a theory, the *Prime Number Theory*, to help predict the number of prime numbers below a given integer (The function is denoted as  $\pi(x)$ , but is not related to the number  $\pi$ ). This theory is remarkable because it shows that  $e$  is indirectly connected to the prime numbers. What is  $\pi(x)$  equivalent to

- A)  $\pi(x) \sim 2\sqrt{\ln x^2}$       B)  $\pi(x) \sim x \ln x$       C)  $\pi(x) \sim \frac{x}{\ln x}$       D)  $\pi(x) \sim \frac{x}{e^2}$       E) NOTA

19) What technology company recently released  $e$  times \$1 billion worth of shares of stock, alluding to the company's many ties to mathematics?

- A) Apple      B) Yahoo!      C) Google      D) Microsoft      E) NOTA

20) What is the nature of the graph of  $f(x) = x^2 + e^{-x}$  at  $x = 1$ ?

- A) increasing, concave up      B) increasing, concave down      E) NOTA  
C) decreasing, concave up      D) decreasing, concave down

21) One value of  $i^i$  is:

- A)  $e^{-\frac{\pi}{4}}$       B) Does Not Exist      C)  $e^{\frac{\pi}{4}}$       D) 1      E) NOTA

22)  $\sinh(x) \cosh(x)$  is equivalent to:

- A)  $4e^{x^2}$       B)  $2e^{2x} - 2e^{-2x}$       C)  $4(e^{x^2} - e^{-x^2})$       D)  $8e^{2x} - 8e^{-2x}$       E) NOTA

23)  $-3 - 3i =$

- A)  $9 \operatorname{cis}\left(\frac{\pi}{4}\right)$       B)  $3\sqrt{2}e^{i\frac{\pi}{4}}$       C)  $9 \operatorname{cis}\left(\frac{\pi}{4}\right)$       D)  $3\sqrt{2}e^{i\frac{5\pi}{4}}$       E) NOTA

24) Solve for  $x$ :  $3x(\ln\sqrt[3]{e^{16}})^x = 192$

- A) 4      B) 3      C)  $\ln 4$       D)  $e^{2x}$       E) NOTA

25) Find  $F'(x)$  when  $F(x) = x^{\frac{1}{\ln x}}$

- A)  $x \ln x$       B) 1      C) 0      D)  $x^{\ln x}$       E) NOTA

26) On the way out to California, Dr. Morris had to catch a connecting flight in the Las Vegas airport. He decided to play a slot machine in the airport with a one in  $n$  probability of winning and he plays it  $n$  times. If he plays an infinite amount of times, what is the probability that he wins nothing at all?

- A) 0      B) .25      C)  $e$       D)  $\frac{1}{e}$       E) NOTA

27) Solve for  $x$ :  $\ln(x^2 - 1) + 1 = \ln(x^2 + 1)$

- A)  $x = \sqrt{\frac{1+e^{-1}}{1-e^{-1}}}$       B)  $x = 1 + e^{-1}$       C)  $x = \frac{1+e^{-1}}{1-e^{-1}}$       D)  $x = \pm \sqrt{\frac{1+e^{-1}}{1-e^{-1}}}$       E) NOTA

28) Which of the following phrases represent the first 10 digits of  $e$ ?

- A) "To oil an car always have some oil handy"      B) "It enables a numskull to memorize a quantity of numerals"      E) NOTA  
 C) "May I have a large container of coffee ready for today?"      D) "My very educated mom just sent us nine pizzas"

29) The nautilus shell, sunflower and some spiral galaxies are examples of what type of spiral?

- A) Euler's Spiral      B) Logarithmic Spiral      C) Hyperbolic Spiral      D) Fermat Spiral      E) NOTA

30) Find  $\lim_{x \rightarrow \infty} x^4 e^{-x}$

- A)  $\infty$       B) Does Not Exist      C)  $-\infty$       D) 0      E) NOTA