

1. C
2. C
3. A
4. A
5. D
6. B
7. B
8. A
9. C
10. C
11. A
12. A
13. B
14. D
15. B
16. A
17. D
18. C
19. C
20. A
21. E
22. B
23. D
24. A
25. C
26. D
27. D
28. B
29. B
30. D

1) C If  $F$  has an inverse, then  $F(F^{-1}(x)) = x$ . So,  $e^{[F^{-1}(x)]^2} = x$  and  
 $\ln x = [F^{-1}(x)]^2$  and  $F^{-1}(x) = \sqrt{\ln x}$ .

2) C The Taylor Series of  $e^{-2x^2}$  center at  $x=1$  is  $F(x) = 1 - 2x^2 + 2x^4 - \frac{4x^6}{3} \dots$ , which is equal to the summation:  $\sum_{n=0}^{\infty} \frac{(-1)^n (2x^2)^{2n}}{n!}$ .

3) A Jacob Bernoulli is credited with the discovery of the constant  $e$  while trying to find the value of the  $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$ , which turns out to be the value  $e$ .

4) A  $e^{2 \ln(4x) - \ln(2x) + \ln(2x)} = e^{\ln(24x^2) - \ln(2x) + \ln(2x)} = e^{\ln 72x^2 / 2x} = \frac{72x^2}{2x}$

5) D Sub in  $x = 2u$  to change the problem to  $\lim_{u \rightarrow \infty} \left(1 + \frac{1}{u}\right)^{6u} = \left(\lim_{u \rightarrow \infty} \left(1 + \frac{1}{u}\right)^u\right)^6 = (e)^6$

6) B  $\frac{d^2y}{dx^2} = n^2 e^{nx}$ ,  $-6 \frac{dy}{dx} = -6ne^{nx}$ ,  $5y = 5e^{nx} \rightarrow \frac{d^2y}{dx^2} - 6 \frac{dy}{dx} + 5y = n^2 e^{nx} - 6ne^{nx} + 5e^{nx} = 0$   
 $n^2 - 6n + 5 = 0 \rightarrow (n-5)(n-1) = 0 \rightarrow n = 5 \text{ and } 1 \text{ sum} = 6$

7) B  $\frac{\ln |\cos(120^\circ)|^4}{\ln(\sin 30^\circ)} = \frac{\ln(1/2)^4}{\ln 1/2} = \frac{\ln 16}{\ln 2} = \frac{4 \ln 2}{\ln 2} = 4$

8) A # of Faces of a polyhedron + # of Vertices - # of edges = 2

9) C  $x = 3 - 4e^{y+7} \rightarrow x - 3 = -4e^{y+7} \rightarrow \frac{3-x}{4} = e^{y+7} \rightarrow f^{-1}(x) = \ln\left(\frac{3-x}{4}\right) - 7$

10) C  $e^{2x} - 12e^{-x} - 1 = 0 \rightarrow e^x(e^x - 12e^{-x} - 1) = 0 * e^x \rightarrow e^{2x} - e^x - 12 = 0$   
 $(e^x - 4)(e^x + 3) = 0$   $e^x = -3$  can never be true and therefore  $e^x = 4 \rightarrow \ln 4 = x$

11) A We know that  $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$ . A property of limits tells us that the left side of this special limit changes the right side by the following property:  $\lim_{x \rightarrow \infty} \left(1 + \frac{b}{x}\right)^{x/c} = e^{b/c}$ , and therefore  $b = -1$ . The limit equals  $e^{-1} = 1/e$

12) A 2.71828182845904523(5)3602875

13) B Use Integration By Parts:  $u = \ln x$   $dv = x^2 dx$

$$du = \frac{1}{x} dx \quad v = \frac{x^3}{3} dx$$

$$\int x^2 \ln(x) dx = \frac{x^3}{3} \ln x - \int \frac{x^2}{3} dx = \frac{x^3}{3} \ln x - \frac{x^3}{9} + C$$

14) D Using Newton's Law of Cooling:

$T(t) = T_s + D_0 e^{-kt}$  with  $T_s = 20$ ,  $D_0 = 100 - 20 = 80$ . Therefore,  $T(t) = 20 + 80e^{-kt}$

Since we know  $T(15) = 75 > 20 \mid 80e^{-15k} = 75 \rightarrow k = \frac{1}{15} \ln(11/16)$ .

And then we substitute in:  $T(25) = 20 + 80e^{(25/15) \ln(11/16)}$

15) B

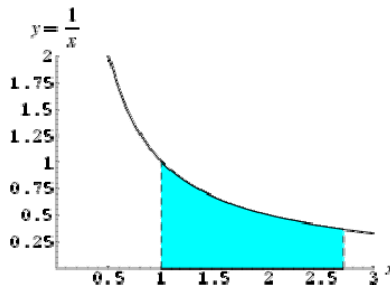
$$\ln(2x - 1) + \ln(3x + 1) = 1 \rightarrow \ln(6x^2 - x - 1) = 1 \rightarrow 6x^2 - x - 1 = e \rightarrow 6x^2 - x + (-1 - e) = 0$$

Using the quadratic formula:  $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{1 - 4(6)(-1-e)}}{12} = \frac{1 \pm \sqrt{25 + 24e}}{12}$ , but can only be the "+" because the negative creates a negative value inside one of the  $\ln()$  and therefore does not work.

16) A Since we take  $\frac{1}{\infty} = 0$  and  $\frac{1}{0} = \infty$ , we can switch around  $\lim_{x \rightarrow 0} (1-x)^{\frac{1}{x}} = \lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^x$  which we know from question #11 to be  $\frac{1}{e}$ .

17) D

$$\int_1^e \frac{dx}{x} = \ln e = 1$$



18) C By definition

19) C On April 29, 2004, Google filed with the SEC to raise \$2,718,281,828 t

20) A Both the 1<sup>st</sup> and 2<sup>nd</sup> derivatives are positive21) E  $i^i = e^{-\left(\frac{\pi}{2} + 2k\pi\right)}$  with  $k = 0, \pm 1, \pm 2, \dots$ 22) B  $\sinh(x) \cosh(x) = e^{\frac{1}{2}x} \cdot \frac{1}{2} \cdot (e^x - e^{-x}) = \frac{1}{2}(e^{2x} - e^{-2x}) = \frac{1}{2}(e^{2x} - e^{-2x})$ 23) D  $-3-3i$   $z = r \operatorname{cis} \theta$   $r = \sqrt{3^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$ 

$$\tan \theta = \frac{b}{a} = 1 \rightarrow \theta = \frac{5\pi}{4} \text{ since in 3rd quadrant } -3 - 3i = 3\sqrt{2} \operatorname{cis} \frac{5\pi}{4} = 3\sqrt{2} e^{i5\pi/4}$$

24) A  $3x(\ln(e^{16})^x) = 3x(\ln e^{2^x}) = 3x(2^x \ln 2) = 192 \rightarrow x2^x = 64 \rightarrow x = 4$ 25) C  $y = x^{\frac{1}{\ln x}} \rightarrow \ln y = \frac{1}{\ln x} \ln x \rightarrow \ln y = 1 \rightarrow y = e^{-1}$  and a derivative of a constant is equal to 026) D  $\lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right)^n = \frac{1}{e}$ 

27) D  $\ln(x^2 - 1) - \ln(x^2 + 1) = -1 \rightarrow \ln \frac{x^2 - 1}{x^2 + 1} = -1 \rightarrow \frac{x^2 - 1}{x^2 + 1} = e^{-1} \rightarrow x^2 - 1 = e^{-1}x^2 + e^{-1}$   
 $x^2 - e^{-1}x^2 = 1 + e^{-1} \rightarrow x^2 = \frac{1 + e^{-1}}{1 - e^{-1}} \rightarrow x = \pm \sqrt{\frac{1 + e^{-1}}{1 - e^{-1}}}$

28) B Count the number of letters in each word: It (2) enables (7) a (1) numskull (8) to (2) memorize (8) a (1) quantity (8) of (2) numerals (8) = 2.718281828

29) B All examples of the Logarithmic Spiral

30) D Using L'Hospital's rule:  $\lim_{x \rightarrow \infty} x^4 e^{-x} = \lim_{x \rightarrow \infty} \frac{4x^3}{e^x} = \frac{12x^2}{e^x} = \frac{24x}{e^x} = \frac{24}{e^x} = \frac{0}{e^x}$