ALPHA BOWL SOLUTIONS NATIONALS 2008

1. $f(x)=\dfrac{(x-4)(x+1)}{(2x-3)(x+1)}=\dfrac{x-4}{2x-3},\ x\neq -1$.

Since there is a hole at $x=-1$, $A=\dfrac{(-1)-4}{2(-1)-3}=1$.

B= $f(4)=0$.

Since there is an asymptote at $x=1.5$, either sketch a graph or plug in a number very close to (but greater than) 1.5 to find that C is $-\infty$.

To find the horizontal asymptote $y=D$, divide the leading coefficients. $D=\dfrac{1}{2}$.

$g(\mathrm{A})+2g(\mathrm{B})+3g(\mathrm{C})+4g(\mathrm{D})=1+0-15+2=-12$

2. $A=-\dfrac{2}{3}$, $B=2$, $C=\dfrac{4}{3}$. $\dfrac{AB}{C}=-1$.

3. $A=3$, $B=\pi$, $C=1$ $ABC=3\pi$.

4. $A=\dfrac{(12)(8)+(5)(6)}{(13)(10)}=\dfrac{63}{65}$. $B=\dfrac{9}{40}$, $C=\sqrt{3}$, $D=\dfrac{\sqrt{5}}{2}$. $\dfrac{AC^2}{BD^2}=\dfrac{672}{65}$

5. $A=\left(\dfrac{4}{10}\right)\left(\dfrac{3}{9}\right)=\dfrac{2}{15}$. $B=$ prob(white, then black) + prob(black, then white) = $\left(\dfrac{4}{10}\right)\left(\dfrac{6}{10}\right)+\left(\dfrac{6}{10}\right)\left(\dfrac{4}{9}\right)=\dfrac{38}{75}$.

C= prob(white sock, then rock) + prob(black sock, then rock) = $\left(\dfrac{3}{10}\right)\left(\dfrac{3}{10}\right)+\left(\dfrac{4}{10}\right)\left(\dfrac{3}{9}\right)=\dfrac{67}{300}$.

$D=\left(\dfrac{3}{10}\right)\left(\dfrac{3}{10}\right)=\dfrac{9}{100}$. $A+B+C+D=\dfrac{143}{150}$

6. $m=8$, $n=-28$. Solving $10-4p=0$, $p=\dfrac{5}{2}$. Solving $\dfrac{-1}{5}=\dfrac{5}{t}$, $t=-25$. $\dfrac{n}{m}-\dfrac{t}{p}=\dfrac{13}{2}$

7. First, $\ln 2=\dfrac{\log 2}{\log e}=\dfrac{\log 2}{\dfrac{\ln e}{\ln 10}}=\dfrac{\dfrac{3}{10}}{\dfrac{1}{\dfrac{23}{10}}}=\dfrac{69}{100}$.

$\log\dfrac{1}{4}=-2\log 2=-0.6$

$\ln 50=\ln 100-\ln 2=2\ln 10-\ln 2=4.6-0.69=3.91$

$\log\dfrac{1}{4}-\ln 50=3.31$.

8. Keep dividing 6240 by 5, and add the integral quotients until the quotient is zero. A=1307

$6240=(2^5)(3^1)(5^1)(13^1)$, so by the counting principle B=$(5+1)(1+1)(1+1)(1+1)=48$

Starting with $80^2=6400$, use trial and error to find that C=6084.

$\text{D}=\left(\frac{1}{4}\right)\left(\frac{11}{10}\right)(6240)=1716$. $A+B+C+D=9155$

9. By the law of sines, $\frac{A}{\sin 100°}=\frac{14}{\sin 50°}$, so $A=\frac{441}{25}$.

By law of sines, $\frac{B}{\sin 30°}=\frac{14}{\sin 50°}$, so $B=9$.

By the law of cosines, $C^2=100+196-2(10)(14)\cos 50°=\frac{584}{5}$

$25A+B+5C^2=1034$

10. $PQ=\begin{bmatrix}5 & 4\\ -5 & 8\end{bmatrix}$ $P^{-1}=\begin{bmatrix}0.3 & 0.1\\ -0.4 & 0.2\end{bmatrix}$ $Q^T=\begin{bmatrix}1 & -3\\ 2 & 0\end{bmatrix}$. $a+d=6.3+8.2=14.5$

11. Set S contains $\text{cis}\frac{k\pi}{6}$, for all integral multiples of *k*.

So $S\cap T=\left\{\text{cis}\frac{5\pi}{6},\text{cis}\frac{4\pi}{3},\text{cis}\frac{3\pi}{2}\right\}=\left\{-\frac{\sqrt{3}}{2}+\frac{i}{2},-\frac{1}{2}-\frac{i\sqrt{3}}{2},-i\right\}$. Of these elements, only the last two are in set *R*.

12. Solving $3x-1=2x^2-2x-4$, $x=-\frac{1}{2}$ *or* 3. Substituting, the intersection points are $\left(-\frac{1}{2},-\frac{5}{2}\right)$ and $(3,8)$.

The distance between the points is $\frac{7\sqrt{10}}{2}$. $g(x)-f(x)+k=2x^2-5x+(k-3)$. Setting the discriminant equal to zero, $25-4(2)(k-3)=0$, find that B=$k=\frac{49}{8}$. C = $-\frac{40}{9}$ $\frac{9BC\sqrt{10}}{A}=-700$

13. Complete the square to get $(x-6)^2+(y+8)^2=121$. Sa A=121, B=6 (from center (6, -8)) C=10 and E=22. The sum is 159.

14. 531 and 532 are not prime. So the next largest is 523 which is prime. So A=523.
5321 base eight is 2769 base ten. C:D=3:1. So 523+2769+3= 3295.

15. A: $4t-t^2=3$ when t=1 or t=3. So y(1)=1 and twice that is 2.

B: x(2),y(2) = (4, 0) and d=4.
C: at the max of x, we have vertex when t=2 (halfway between roots) and y(2)=0.
D: at t=1 (3,1) and at t=2 (4, 0) gives d=$\sqrt{2}$.
Final: 2+4+0+2 = **8**.