1. A
2. C
3. B
4. C
5. C
6. C
7. D
8. B
9. B
10. B
11. B
12. E
13. E
14. B
15. C
16. E
17. B
18. C
19. A
20. C
21. D
22. C
23. A
24. A
25. C
26. A
27. C
28. E
29. C
30. C

1. Water is running into the tank at $\frac{4}{3}$ gallons each day and running out of the tank at $\frac{2}{4}$ gallons each day. Therefore $\frac{4}{3}-\frac{2}{4}=\frac{5}{6}$ gallons. **Answer choice A.**

2. Let $x=$ the amount of money. Therefore,

$$\begin{aligned}&\frac{1}{4}x+\frac{1}{5}\left(\frac{3}{4}x\right)+66=x\\&5x+3x+1320=20x\\&12x=1320\\&x=110\end{aligned}$$

Answer choice C.

3. Let $x=$ the amount invested at 9%
$2x=$ the mount invested at 6%
$25,000-3x=$ the amount invested at 8%

$$\begin{aligned}&.09x+2(.06x)+.08(25,000-3x)=1850\\&.09x+.12x+2,000-.24x=1850\\&-.03x=-150\\&x=5000\end{aligned}$$

Answer choice B.

4. $\begin{bmatrix}3 & 5\\-2 & 4\\4 & -3\\3 & 5\end{bmatrix}=\left|\frac{-3-38}{2}\right|=20.5\approx 21$. **Answer choice C.**

5. Keychain permutation: $\frac{(n-1)!}{2}$, Divide by 2 again (there are two R's), $\frac{\frac{(6-1)!}{2}}{2}=\boxed{30}$. **Answer choice C**.

6. By graphing and getting the intersection points, we have a trapezoid with height 4 and bases 5 and 11. Therefore the area is 32. **Answer choice C**.

7. When finding the greatest distance one must consider three points: the two bounds and the vertex. When plugging these values back into $x(t)$ we find that when $t=4$, the particle is 62 units away, hence the greatest distance. **Answer choice D.**

8. $J = 3R$
$J - 1 = 4(R-1)$
$J = 9, R = 3$
$J = 12, R = 6$
Answer choice B.

9. $100 = \frac{4}{3}\pi r^3 + \pi r^2 h \Rightarrow 100 - \frac{4}{3}\pi r^3 = \pi r^2 h \Rightarrow h = \frac{300 - 4\pi r^3}{3\pi r^2}$.**Answer choice B**

10. Bill's rate is $\frac{1}{8}$ of a sign per hour. Bob's rate is $\frac{1}{4}$ of a sign per hour. Barry's rate is $\frac{1}{6}$ of a sign per hour. Therefore,

$\frac{1}{6}(3) + \frac{1}{4}\left(\frac{7}{2}\right) + \frac{1}{8}\left(x + \frac{7}{2}\right) = 2 \to \frac{1}{8}x = \frac{3}{16} \to \frac{3}{2}$ hours. **Answer choice B.**

11. $\frac{156}{r} - \frac{3}{4} = \frac{156}{r+9} \to r^2 + 9r - 1872 = 0 \to (r+48)(r-39) = 0 \to r = 39$ **Answer choice B.**

12. Let $x =$ original cost. Therefore $\left(\frac{3}{2}\right)\left(\frac{4}{5}\right)\left(\frac{4}{5}\right)x = 5.28 \to \frac{24}{25}x = 5.28 \to x = 5.50$. **Answer E.**

13. $\sqrt{\left(100^2 + x^2\right)} = \sqrt{\left(60^2 + \left(200 - x^2\right)\right)}$
$10000 + x^2 = 3600 + 40000 - 400x + x^2 \to x = 84$
Answer Choice E.

14. Solving the equations $4c + 7a = 165$ and $a + c = 27$ gives c=8. **Answer Choice B**

15. $\frac{90 + 90\left(\frac{2}{3}\right)}{1 - \frac{2}{3}} = \frac{150}{\frac{1}{3}} = 450$. **Answer choice C.**

16. We are essentially looking for 2 to what power is equal to 32768. Continuously dividing by 2 and counting the number of times, you will get 15. **Answer choice E**.

17. There are $2n-1$ people in the n^{th} row. Since we are dealing with the sum of the odd integers n^2, we find $n^2 = 1600 \to n = 40$. Now, plugging in 40 we get 39. **Answer choice B**.

18. 60% of the coins are gold and 80% of the objects are coins. Therefore $(.6)(.8) = .48$ or 48% are gold coins. **Answer choice C.**

19. $\frac{Rd^2}{1} = k \rightarrow \frac{(10)(3^2)}{65} = \frac{R(5^2)}{50} \rightarrow R = \frac{180}{65} = \frac{36}{13}$ **Answer choice A.**

20.
$$(x)^2 + (x+1)^2 + (x+2)^2 = 434$$
$$x^2 + x^2 + 2x + 1 + x^2 + 4x + 4 = 434$$
$$3x^2 + 6x + 5 = 434$$
$$3x^2 + 6x - 429 = 0$$
$$x^2 + 2x - 143 = 0$$
$$(x+13)(x-11) = 0$$
$$x = -13, x = 11$$
$$-13 - 12 - 11 = -36$$

Answer choice C.

21. $k_1 = 2x^2 - 4x - k = 0 \Rightarrow (-4)^2 - 4(2)(-k) = 0 \Rightarrow 16 + 8k = 0 \Rightarrow 8k = -16 \Rightarrow k = -2$

$k_2 = 2(10)^2 - 4(10) - k = 0 \Rightarrow 200 - 40 - k = 0 \Rightarrow k = 160$

$k_3 = 2(x+10)^2 - 4(x+10) - k = 0 \Rightarrow 2x^2 + 40x + 200 - 4x - 40 - k = 0$

$2x^2 + 36x + 160 - k = 0 \Rightarrow 2x^2 + 36x + 160 - 2x^2 + 4x = 0 \Rightarrow 40x + 160 = 0 \Rightarrow x = -4$

$\Rightarrow 2(-4)^2 - 4(-4) - k = 0 \Rightarrow 32 + 16 = k \Rightarrow k = 48$

-2 + 160 + 48 = 206 **Answer Choice D.**

22. Since the equation has a cubed x term and a cubed constant term (216), we can find a factor of $x^3 + kx^2 - 54x + 216 = 0$ by using this information. Since both the x cubed term and the 216 are positive, according to the rule of factoring cubes, we use the cube root of x cubed and the cube root of 216 as a factor $(x+6)$. Using this as a factor, we can use -6 as a root. Using synthetic division, we get the quadratic $x^2 - 15x + 36$. Factoring this quadratic we get $(x-3)(x-12)$. So, now we have $(x-3)(x+6)(x-12)$= 0. The roots are in a geometric progression with a common ratio of -2. The value of k is the sum of the roots which is 3 + 12 – 6 = 9. Since k is positive, we must take the opposite of the answer, which is – 9. **Answer choice C.**

23.
$$6x + 8y + 31 + 4y^2 - 6x - 8y - 67 = 0$$
$$4y^2 - 36 = 0 \Rightarrow y^2 - 9 = 0 \Rightarrow y = \pm 3$$

Since we have two solutions, we will get four points, which are the vertices of the trapezoid. Plugging the values of y into either of the equations we get the points (3,11), (3, -5), (-3, 7), and (-3,-1). Using the points to find the area, we find the area to be 72. **Answer choice A.**

24. If a complex number in the form of $a+bi$, where $a,b \neq 0$, the complex number, multiplied by its conjugate will always be a real, positive number. **Answer choice A.**

25. The sample space is the number of ways the 8 people can be situated around the table which is 7!. The number of satisfying situations can be found by initially treating the couples as single people. That gives 5! arrangements which is multiplied by four as each couple can be seated in two ways. $=\frac{4*5!}{7!}=\frac{2}{21}$. **B**

26. $\sqrt{ab} \geq \frac{a+b}{2} \Rightarrow ab \geq \frac{(a+b)^2}{4} \Rightarrow 0 \geq \frac{(a+b)^2}{4} - \frac{4ab}{4}$

$0 \geq \frac{(a-b)^2}{4}$

So, evaluating each case using the above formula we find that

I. false when $a=b=1$
II. false when $a=b=2$
III. false when $a=b=1$
IV. false when $a=b=2$

Therefore the **answer choice is A.** (The question says which of the following must *always be true*.)

27. $180(n-2)=88560 \Rightarrow 180n-360=88560 \Rightarrow 180n=88920 \Rightarrow n=494$. **Answer choice C.**

28. If the equilateral triangle has one side on the x-axis, the slopes of the sides of the equilateral triangle will equal 0. **Answer choice E.**

29. The number of ways you can choose a 5 letter word with a vowel in it is ${}_8C_2=28$. Of these combinations, you can have a 5 letter word with either of the two vowels (this is 2 combinations) or both vowels (this is 1 combination). To find the probability that the word contains at least one vowel, do $1-\frac{3}{28}=\frac{25}{28}$. **Answer choice C.**

30. The number of the students who played only sports is equal to $45-20-7-1=17$. the number of students who played only musical instruments is equal to $34-20-6-1=7$. the number of students who participated only on academic teams is equal to $15-7-6-1=1$. Therefore, the number who did not do any of the three activities is equal to $100-17-7-1-20-6-7-1=41$ students. **Answer choice C**.