



1. A
2. C
3. E
4. C
5. B
6. E
7. D
8. D
9. D
10. C
11. A
12. D
13. C
14. B
15. D
16. C
17. B
18. A
19. D
20. B
21. B
22. A
23. C
24. B
25. C
26. D
27. B
28. C
29. C
30. C



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1.  $(\sqrt[3]{9})(\sqrt[4]{16})(\sqrt[6]{36}) = (3^{\frac{2}{3}})(2)(6^{\frac{1}{3}}) = 2\sqrt[3]{54} = 6\sqrt[3]{2} \Rightarrow 6+3+2=11 \Rightarrow A$

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2.  $f(x+3)$  and  $g(x+3)$  shifts  $f(x)$  and  $g(x)$  each 3 units left, so  $(5,1)$  shifts to  $(2,1) \Rightarrow C$

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$$1 - \frac{1}{1-x} = \frac{1-x-1}{1-x} = \frac{-x}{1-x} = \frac{x}{x-1}$$

3.  $\frac{1}{\frac{x}{x-1}} = \frac{x-1}{x} \Rightarrow \frac{1}{1-\frac{x-1}{x}} = \frac{x}{x-x+1} = x \Rightarrow E$

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4.  $i^{4n+1} = i$ , not  $-i \Rightarrow C$  is false

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$$x + \sqrt{x-4} = 6 \Rightarrow \sqrt{x-4} = 6-x \Rightarrow x-4 = 36-12x+x^2$$

5.  $0 = x^2 - 13x + 40 \Rightarrow 0 = (x-8)(x-5) \Rightarrow x=8$  or  $x=5$

$8 + \sqrt{4} \neq 6$ , so 8 is extraneous ;  $5 + \sqrt{1} = 6$ , so 5 is the only root  $\Rightarrow B$

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$$f(2) = f(1) + 2 = 3$$

$$f(3) = f(2) + 3 = 6 \quad \therefore f(4) + f(5) = 25 \Rightarrow C$$

6.  $f(4) = f(3) + 4 = 10$

$$f(5) = f(4) + 5 = 15$$

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7. By the rational root theorem,  $\frac{3}{2}$  is not a possible rational root because 2 is not a factor of the leading coefficient, 9.  $\Rightarrow D$

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8.  $P(\text{pass at least one}) = 1 - P(\text{pass neither}) = 1 - (0.7)(0.4) = 0.72 \Rightarrow D$

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The units digit of  $2^n$  ( $n \in \{\text{Natural Numbers}\}$ ) repeats in a cycle of 4  $\Rightarrow 2, 4, 8, 6, 2, 4, \dots$

9. and the units digit of  $3^n$  also repeats in a cycle of 4  $\Rightarrow 7, 9, 3, 1, 7, 9, \dots$

Since  $2006 \div 4$  leaves a remainder of 2,  $X=4$  and  $Y=9$ , so  $XY=36$ .  $\Rightarrow D$

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10.  $W = \frac{kX}{Y^2}$  so when X and Y are both increased by 25%  $\Rightarrow W = \frac{k(1.25X)}{(1.25Y)^2}$ ,

which reduces to  $W = \frac{kX}{1.25Y^2} = \frac{4}{5} \bullet \frac{kX}{Y^2} \Rightarrow W$  is decreased by 20%  $\Rightarrow C$

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11.  $2^{-n} \cdot 8^{n-1} \cdot 4^{n+3} \div 16^n = 2^{-n} \cdot 2^{3n-3} \cdot 2^{2n+6} \div 2^{4n} = 2^3 = 8 \Rightarrow A$

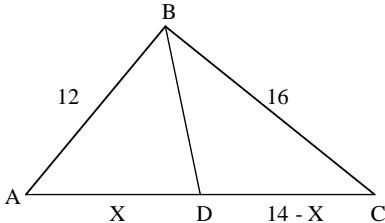
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12.  $\frac{\frac{b}{a} - k}{\frac{1}{b} - \frac{1}{a}} = -b$ , so multiplying both sides by  $\frac{1}{b} - \frac{1}{a} \Rightarrow -1 + \frac{b}{a} = \frac{b}{a} - k \Rightarrow k = 1 \Rightarrow D$

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By the angle bisector theorem:

13.  $\frac{X}{12} = \frac{14-X}{16}$   
 $16X = 168 - 12X$   
 $28X = 168 \Rightarrow X = 6 \Rightarrow C$



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14.  $(\log_p x^3)(\log_4 p) = 6 \Rightarrow \frac{\log x^3}{\log p} \cdot \frac{\log p}{\log 4} = 6 \Rightarrow \frac{\log x^3}{\log 4} = 6 \Rightarrow \log_4 x^3 = 6$   
 $\Rightarrow x^3 = 4^6 \Rightarrow x = 16 \Rightarrow B$

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$x^3 + x^2 - 9x - 9 = 0 \Rightarrow (x+3)(x-3)(x+1) = 0 \Rightarrow x = -3 \text{ or } 3 \text{ or } -1 \Rightarrow A = 3$

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$\frac{12}{x-1} - \frac{8}{x} = 2 \quad (x \neq 1 \text{ or } 0) \Rightarrow 12x - 8(x-1) = 2x(x-1) \Rightarrow x^2 - 3x - 4 = 0$   
 $\Rightarrow x = 4 \text{ or } -1 \Rightarrow B = 3$

15. 

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Let  $y = x^2 - 1 \Rightarrow y^2 - 5y + 4 = 0 \Rightarrow (y-4)(y-1) = 0$   
 $\Rightarrow x^2 - 1 = 4 \text{ or } 1 \Rightarrow x^2 = 5 \text{ or } 2 \Rightarrow x = \sqrt{5} \text{ or } -\sqrt{5} \text{ or } \sqrt{2} \text{ or } -\sqrt{2} \Rightarrow C = 10$

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$A + B + C = 16 \Rightarrow D$

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16.  $8(P) \cdot 6(VP) \cdot 12(S) = 576 \Rightarrow C$

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Let the roots of  $x^2 + 4x + 5 = 0$  be p and q. Then,  $p + q = -4$  and  $pq = 5$ .

17.  $p^2q^2 = (pq)^2 = c = 25$   
 $p^2 + q^2 = (p+q)^2 - 2pq = -b = (-4)^2 - 2(5) = 6 \Rightarrow b = -6$   
 $b+c=19 \Rightarrow B$

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$x + y - 10 = 0 \Rightarrow A = 1, B = 1, C = -10$  and  $y = -x + 7$  contains  $(0, 7) \Rightarrow x_1 = 0$  and  $y_1 = 7$

18.  $d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}} = \frac{|0 + 7 - 10|}{\sqrt{1+1}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2} \Rightarrow A$

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19.  ${}_9P_2 = \frac{9!}{7!} = 72 \quad | \quad \begin{vmatrix} 4 & 2 \\ 10 & 5 \end{vmatrix} = 20 - 20 = 0 \quad | \quad C_2 = \frac{4!}{2!2!} = 6 \quad | \quad 3! = 6$   
 $X = \frac{72}{0+6+6} = 6 \Rightarrow 2^6 = 64 \Rightarrow D$

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This equation will be true if  $x^2 - 9x + 19 = 1 \Rightarrow (x-3)(x-6) = 0 \Rightarrow x = 3$  or  $6$

20. or  $x^2 + 2x - 3 = 0$  (and  $x^2 - 9x + 19 \neq 0$ )  $\Rightarrow (x+3)(x-1) = 0 \Rightarrow x = -3$  or  $1$   
or  $x^2 - 9x + 19 = -1$  (and  $x^2 + 2x - 3$  is even)  $\Rightarrow (x-5)(x-4) = 0 \Rightarrow x = 5$  or  $4$   
 $5^2 + 2(5) - 3$  is even, but  $4^2 + 2(4) - 3$  is odd, so  $4$  is extraneous  $\Rightarrow 3+6-3+1+5=12 \Rightarrow B$

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21.  $3x + 2yi = -63 - 54i \Rightarrow 3x = -63$  and  $2y = -54 \Rightarrow x = -21$  and  $y = -27$   
 $\Rightarrow x + y = -48 \Rightarrow B$

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The parabola opens right with vertex  $(1, 2) \Rightarrow p = 2 \Rightarrow 4p = 8 \Rightarrow |a| = \frac{1}{8} \Rightarrow a = \frac{1}{8}$

22. Vertex form:  $x - h = a(y - k)^2 \Rightarrow x - 1 = \frac{1}{8}(y - 2)^2 \Rightarrow 8x - 8 = y^2 - 4y + 4$   
 $\Rightarrow 0 = y^2 - 8x - 4y + 12 \Rightarrow A$

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23.  $q^{52} = q^{-4}r^7 \Rightarrow q^{56} = r^7 \Rightarrow r = q^8$  so  $q^{-4} \bullet q^8 = q^p \Rightarrow q^4 = q^p \Rightarrow p = 4 \Rightarrow C$

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24.  $kx^2 + 8x + 3 = 0$  will have imaginary roots if  $b^2 - 4ac < 0 \Rightarrow 64 - 12k < 0 \Rightarrow k > \frac{16}{3} \Rightarrow B$

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25. i is false (Domain =  $\{x : x > 0\}$ ); ii is false (strictly increasing); iii, iv, and v are all true  $\Rightarrow C$

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Let measures of exterior angle =  $x$  and interior angle =  $5x \Rightarrow 6x = 180 \Rightarrow x = 30$  and  $5x = 150$

26.  $\frac{(n-2) \cdot 180}{n} = 150 \Rightarrow 180n - 360 = 150n \Rightarrow n = 12 \Rightarrow 12(150) = 1800 \Rightarrow D$

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$$25(x^2 - 2x + 1) + 9(y^2 + 4y + 4) = 164 + 25 + 36 \Rightarrow \frac{(x-1)^2}{9} + \frac{(y+2)^2}{25} = 1$$

27. Center is  $(1, -2)$  and major axis is vertical  $\Rightarrow c^2 = 25 - 9 \Rightarrow c = 4 \Rightarrow$  foci are  $(1, 2)$  and  $(1, -6)$   
Sum of  $y$ -coordinates =  $-4 \Rightarrow B$
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$$\sum_{x=1}^3 (ax + b) = 15 \Rightarrow a + b + 2a + b + 3a + b = 15 \Rightarrow 6a + 3b = 15 \Rightarrow 2a + b = 5$$

28.  $\sum_{x=2}^4 (ax + b) = 21 \Rightarrow 2a + b + 3a + b + 4a + b = 21 \Rightarrow 9a + 3b = 21 \Rightarrow 3a + b = 7$

Solving the system  $\Rightarrow a = 2$  and  $b = 1 \Rightarrow a + b = 3 \Rightarrow C$

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29.  $f(x) = x^4 + 5x^2 + 2x - 11$  has 1 sign change  $\Rightarrow A = 1$

$$f(-x) = x^4 + 5x^2 - 2x - 11$$
 has 1 sign change  $\Rightarrow B = 1 \Rightarrow A - B = 0 \Rightarrow C$

30.  $f(x) = \frac{(x-5)(x+3)}{(x+3)(x-2)}$  i is false (vertical asymptote is  $x = 2$ ); ii is true;

iii is false (horizontal asymptote is  $y = 1$ ); iv is true (5 is the real zero)  $\Rightarrow C$