1. 8
2. 3/2 (also accept 1.5 or 1 ½)
3. $x\ln(x) - x + C$.
4. 6
5. 11
6. –2/3
7. 4
8. Tautology
9. 31,680
10. 1
11. 30
12. 10π
13. 360/7
14. 3
15. 2
16. $(2x - y)(3x - z)$
17. 3
18. 36π
19. 12
20. 3
21. 5/3
22. 15/4 (also accept 3.75 or 3 ¾)
23. (7)(11)(13)
24. DNE
25. 6

1. 8 The formula for the sum of roots (which *does* account for double roots) = -B/A, where in this case B = -8 and A = 1.

2. 3/2 Factor by completing the square to attain the standard form of the hyperbola, which will have values $a^2 = 4$ and $b^2 = 5$. Hence $c^2 = a^2 + b^2 = 9$, and finally eccentricity = c/a = 3/2 (also accept 1.5 or 1½).

3. $x\ln(x) - x + C$.
Using the integration by parts formula with $u = \ln(x)$ and $dv = dx$, we arrive without much computation at $x\ln(x) - x + C$.

4. 6 Draw three diagonals coming from the same vertex, and a fourth diagonal that intersects only one of the first three.

5. 11 The number of total marbles is less than 50, is divisible by three, and is equivalent to 1 (mod 4). This yields the possibilities 9, 21, 33, and 45. The number is also equivalent to 3 (mod 5), which is only satisfied by 33. Hence, each child has 11 marbles. (This type of problem can be computed algorithmically using a method of number theory called the Chinese Remainder Theorem.)

6. –2/3 The slope of the tangent line at $x = 1/9$ is:

$$f'(1/9) = (1/2)(1/9)^{-(1/2)} = \frac{1}{2\sqrt{1/9}} = \frac{1}{2/3} = 3/2.$$

The slope of the normal line is hence the opposite reciprocal, -2/3.

7. 4 Unlike quadratic, cubic, and quartic polynomials, the general quintic cannot be solved algebraically using a finite number of additions, subtractions, multiplications, divisions, and root extractions. This is a famous and difficult result that was proved by Abel (Abel's impossibility theorem) and Galois.

8. Tautology
This is the book definition of a tautology.

9. 31,680 We are choosing 1 ace out of 4 total, 3 face cards out of 12 total, and 1 non-ace/non-face card out of 36 total. This is given by $({}_4C_1)({}_{12}C_3)({}_{36}C_1) = 31{,}680$.

10. 1 Using L'Hospital's rule, we get: $\lim_{n\to 0} \frac{1 - n/\sqrt{n^2+1}}{-\sin(n) + \cos(n)} = 1$.

11. 30 Square pyramidal numbers can be represented by stacking diminishing square grids of balls on top of one another to form a pyramid, hence the nth square pyramidal number is the sum of the first n square numbers. The fourth square pyramidal number is then 1 + 4 + 9 + 16 = 30.

12. 10π Frequency and period, which unless otherwise specified are independent of amplitude, obey the relationship $fp = 2\pi$, hence $(1/5)p = 2\pi$, and $p = 10\pi$.

13. 360/7 Let the angle measures be represented by x, 2x, and 4x. Then, since the angle measure total of a triangle must equal 180, let x + 2x + 4x = 180. We see that x = 180/7 and so the second largest angle will be 360/7.

14. 3 These two functions will intersect once in the second quadrant (clearly by observing graph behavior) and twice in the first quadrant (not as obvious). It is not too difficult to see that the functions will intersect at $x = 2$, but the third intersection, which occurs further to the right, cannot be directly computed. Instead, we observe that $f(x)$ is greater than $g(x)$ to the right of $x = 2$, yet $g(x)$ grows much faster than $f(x)$ and must therefore at some point overtake $f(x)$.

15. 2 Notice that $g(x)$ will converge to the line $y = 0$ over the interval in question, so we really only need to integrate $f(x)$. Doing so yields an area of 2.

16. $(2x-y)(3x-z)$
We can use factoring by grouping to get $3x(2x-y) - z(2x-y)$ which can then be further factored into $(2x-y)(3x-z)$.

17. 3 The graph will cross over the axis at single roots (as well as any root that occurs an odd number of times), and will "kiss" the axis without crossing it at double roots (as well as any root that occurs an even number of times).

18. 36π The old formula still holds; area = height * base area = $4 * 9\pi = 36\pi$.

19. 12 The coefficient, n, of theta is even, so there are $2n$ petals.

20. 3 There are three: the tetrahedron, the octahedron, and the icosahedron.

21. 5/3 The probability of throwing a 1 is 1/3, so if the die is thrown repeatedly, a 1 should be rolled 1/3 of the time. There are 5 rolls, so the expected value is 5/3.

22. 15/4 The arithmetic mean is $(9 + 1)/2 = 5$. The geometric mean is $(9*1)^{1/2} = 3$. The harmonic mean of 3 and 5 is $2(3)(5)/(3 + 5) = 30/8 = 15/4$ (also accept 3.75, 3 ¾).

23. (7)(11)(13)
Using divisibility tricks, it can be seen that 1001 is not divisible by any integer less than 7. Seven divides 1001 into 143, which satisfies the divisibility trick for 11: the middle digit equals the sum of the first and third digits, and the first and third digits indicate the second factor, 13.

24. DNE This function will oscillate infinitely violently as x approaches 0. No matter how close to zero you get, you can find points even closer to zero where the function takes on values of both 1 and –1. The limit does not exist.

25. 6 When working under a modulus fractions must be treated as inverses of integers, so ½ is interpreted as "the multiplicative inverse of 2," in other words, the number that when multiplied by 2 will yield 1 (mod 11). Since 2 * 6 = 12, and 12 is congruent to 1 (mod 11), we conclude that 6 is the inverse of 2, modulo 11.