



1. -47
2. $-2/\pi$
3. $2/\pi$
4. -12
5. $-25/2$
6. $-1/20$
7. $\pi/3$
8. $\frac{9\sqrt{3}-6}{\pi}$
9. $1/6$
10. $\frac{\sqrt{2}}{4}$
11. 1
12. b, c, e
13. $-1/5$
14. $3\pi/112$
15. $\frac{3\sqrt{2}}{2}$



1. $a = dv/dt = 2 + 6t$
 $v = ds/dt = 2t + 3t^2 + 3$
 $s = t^2 + t^3 + 3t + 2$
 $A = s(1) = 1 + 1 + 3 + 2 = 7$
 $a = 12, v = 12t, B = \int_0^3 v dt = \int_0^3 12t dt = 6t^2 \Big|_0^3 = 54$
 $A - B = 7 - 54 = -47$

2. $\lim_{x \rightarrow 3} \frac{x-3}{x^2-2x-3} = \lim_{x \rightarrow 3} \frac{1}{x+1} = 1/4$ so $\lim_{x \rightarrow \infty} \frac{4-x^2}{4x^2-x-2} = -1/4$
 $\lim_{x \rightarrow -\infty} \tan^{-1} x = -\pi/2$ so $\lim_{x \rightarrow \infty} \frac{3x^2+27}{x^3-27} = 0$ thus, $\frac{(AB)^D}{C} = \frac{(1/4 \cdot -1/4)^0}{-\pi/2} = -2/\pi$

3. $\mathbf{R} = 3\cos(\pi/3)\mathbf{i} + 2\sin(\pi/3)\mathbf{j}$ $\mathbf{v} = -\pi\sin(\pi/3)\mathbf{i} + (2\pi/3)\cos(\pi/3)\mathbf{j}$
 $\mathbf{a} = (-\pi^2/3)\cos(\pi/3)\mathbf{i} - (2\pi^2/9)\sin(\pi/3)\mathbf{j}$

$$A = v(3) = \sqrt{(-\pi \cdot 0)^2 + \left(\frac{2\pi}{3} \cdot -1\right)^2} = 2\pi/3 \quad B = a(3) = \sqrt{\left(-\frac{\pi^2}{3} \cdot -1\right)^2 + \left(-\frac{2\pi^2}{9} \cdot 0\right)^2} = \pi^2/3$$

$$A/B = 2/\pi$$

4. $K'(1) = \left(\frac{1}{g}\right)'(1) = -1 \cdot \frac{1}{[g(1)]^2} \cdot g'(1)$
 $= -1 \cdot \frac{1}{3^2} \cdot (-3) = 1/3$
 $M'(1) = f'(g(1)) \cdot g'(1) = f'(3)g'(1)$
 $= 4(-3) = -12$
 $[f(x^3)]' = f'(x^3) \cdot 3x^2 \rightarrow P'(1)$
 $= f'(1) \cdot 3 = 2 \cdot 3 = 6$
 $S'(3) = \frac{1}{f'(S(3))} = \frac{1}{f'(f^{-1}(3))} = \frac{1}{f'(1)} = 1/2$
 $ABCD = 1/3 * -12 * 6 * 1/2 = -12$

5. $y = 4 - x^2$
 $dy/dx = -2x = -2(1) = -2$
 $y - 3 = -2(x-1)$
 $y = -2x + 5$ so the intercepts are $(0, 5), (5/2, 0)$ and $\text{Area} = \frac{1}{2}(5)(5/2) = 25/4$

Let $m = \text{slope of line}$ Equation of line: $y - 2 = m(x - 1)$ Intercepts: $(0, 2 - m), (1 - 2/m, 0)$
 $\text{Area} = \frac{1}{2}(2 - m)(1 - 2/m) = \frac{1}{2}(4 - 4/m - m)$
 $dA/dm = \frac{1}{2}(4/m^2 - 1) = 0$ m must be negative, so $m = -2$ $AB = (25/4)(-2) = -25/2$



6. $y = x^5 + x^3 - 2x$
 $y' = 5x^4 + 3x^2 - 2$
 $y'' = 20x^3 + 6x = x(20x^2 + 6) = 0, x = 0$
 $y'(0) = -2$

$y = xe^{-x}, y' = e^{-x}(1 - x) = 0, x = 1$

$y = x^4 - 4x^2$
 $y' = 4x^3 - 8x$
 $y'' = 12x^2 - 8 = 0$

the equation has two roots, and y'' changes sign at both
therefore the number of inflection points is 2

$f(x) = 4\sin x - 3\cos x$
 $f'(x) = 4\cos x + 3\sin x = 0, \tan x = -4/3$
therefore, for the interval $[\pi/2, \pi]$
 $\sin x = -4/5, \cos x = 3/5$
maximum value of $f(x) =$
 $4(4/5) - 3(-3/5) = 5$

$1/(ABCD) = 1/(-2*1*2*5) = -1/20$

7.

$A = \int_{-3}^3 \frac{dx}{9+x^2} = \frac{\tan^{-1}(x/3)}{3} \Big|_{-3}^3$
 $= 1/3 \left(\frac{\pi}{4} - -\frac{\pi}{4} \right) = \pi/6$

$B = \int_1^e \frac{\ln x}{x} dx = \frac{\ln^2 x}{2} \Big|_1^e = \frac{\ln^2 e}{2} - \frac{\ln^2 1}{2} = 1/2$ $C = \int_0^1 xe^x dx = xe^x - e^x \Big|_0^1 = 1$

$A/(BC) = (\pi/6)/(1/2) = \pi/3$

8.

$AV = \frac{1}{\frac{\pi}{2} - \frac{\pi}{3}} \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos x dx = \frac{6}{\pi} \sin x \Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}}$
 $= \frac{6}{\pi} \left(1 - \frac{\sqrt{3}}{2} \right) = \frac{3(2 - \sqrt{3})}{\pi}$

$AV = \frac{1}{\frac{\pi}{4} - \frac{\pi}{6}} \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \csc^2 x dx = -\frac{12}{\pi} \cot x \Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}}$
 $= -\frac{12}{\pi} (1 - \sqrt{3}) = \frac{12(\sqrt{3} - 1)}{\pi}$

$A + B = \frac{9\sqrt{3} - 6}{\pi}$



9. $x = 2\cos\theta$ and $y = 3\sin\theta$.

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$(x/2)^2 + (y/3)^2 = 1$$

$$A = \pi ab = \pi(2)(3) = 6\pi$$

$$A = \int_{-2}^2 \frac{4}{x^2 + 4} dx = 2 \tan^{-1}(x/2) \Big|_{-2}^2 = \pi$$

$$B/A = \pi/6\pi = 1/6$$

10.

$$y \frac{dy}{dx} = x \rightarrow y dy = x dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + c \rightarrow c = 1/2$$

$$f(1) = \sqrt{2}$$

$$(dy/dx)^2 = y$$

$$dy/dx = \sqrt{y} \rightarrow \frac{dy}{\sqrt{y}} = dx$$

$$2\sqrt{y} = x + c \rightarrow c = 0$$

$$g(1) = 1/4$$

$$AB = \frac{\sqrt{2}}{4}$$

11. $A(x) = \ln(\sec x + \tan x)$

$$A'(x) = \frac{\sec x(\tan x + \sec x)}{\sec x + \tan x} = \sec x$$

$$A'(0) = \sec 0 = 1$$

$$B(x) = e^{-x} \cos 2x$$

$$B'(x) = e^{-x}(-2 \sin 2x) + \cos 2x(-e^{-x})$$

$$B'(0) = -1$$

$$C(x) = \sin^{-1} x - \sqrt{1-x^2}$$

$$C'(x) = \frac{1}{\sqrt{1-x^2}} - \frac{1 \cdot (-2x)}{2\sqrt{1-x^2}}$$

$$C'(0) = 1$$

$$A'(0) + B'(0) + C'(0) = 1 + -1 + 1 = 1$$



12. The p series $\sum \frac{1}{n^p}$ converges for $p > 1$ and diverges for $p \leq 1$.

Therefore, b and c diverge, while a, d, and h converge

Alternating series converge if $u_{n+1} < u_n$ and $\lim_{n \rightarrow \infty} u_n = 0$. This condition is satisfied for series (f) and (g).

However, series (e) diverges because $\lim_{n \rightarrow \infty} u_n = 1$.

Thus, the series that diverge are (b), (c), and (e).

13.

$$y = x^3, y' = 3x^2 = 3, x = -1, 1$$

$$y - 1 = 3(x - 1) \text{ or } y + 1 = 3(x + 1)$$

$$y = 3x - 2 \text{ or } y = 3x + 2, k = -2, 2$$

$$y = x^2, y' = 2x$$

Equation of tangent is $y - 5 = 2x(x - 3)$

$$x^2 - 5 = 2x(x - 3), x = 1, 5$$

slope = $2x = 2, 10$

$$(AB)/(CD) = (-2 \cdot 2)/(2 \cdot 10) = -1/5$$

14.

$$A = \pi \int_0^{\pi} y^2 dx = \pi \int_0^{\pi} (\sin x)^2 dx = \pi \int_0^{\pi} \frac{1 - \cos 2x}{2} dx$$

$$= \pi \left(\frac{x}{2} - \frac{\sin 2x}{4} \right) \Big|_0^{\pi} = \pi \left(\frac{\pi}{2} \right) = \pi^2 / 2$$

The equation of the line connecting the points (2, 2) and (4, 4) is $y = x$.

$$B = \pi \int_2^4 y^2 dx = \pi \int_2^4 x^2 dx = \pi \frac{x^3}{3} \Big|_2^4 = \pi \left(\frac{64}{3} - \frac{8}{3} \right) = \frac{56\pi}{3}$$

$$A/B = (\pi^2/2) / (56\pi/3) = 3\pi/112$$

15. $f(x) = \cos(x), f'(x) = -\sin(x), f''(x) = -\cos(x), f'''(x) = \sin(x)$

$$\text{Coefficient} = f'''(\pi/4)/3! = \sin(\pi/4)/6 = \sqrt{2}/12$$

$$y = e^{\sin x}, y' = \cos x (e^{\sin x}), y'' = (e^{\sin x})(\cos^2 x - \sin x)$$

$$\text{Coefficient} = f''(0)/2! = 1/4$$

$$B/A = \frac{1}{4} / \frac{\sqrt{2}}{12} = \frac{3\sqrt{2}}{2}$$