Theta Topic Test – Functions - Solutions
FAMAT State Convention 2004

For all questions, answer "E) NOTA" means none of the above answers is correct.
The figures in this test are not drawn to scale.

1. C. Find that $f(x) = 2x^2 + 7x - 5$ and evaluate f(5) – f(3) = 80-34 = 46.
2. B. Multiply equation 1 by (2) and add to equation 3. Solve equation for y and substitute back in to find that x = 69, y = -125, z = -49. Thus x+ y + z = -105
3. D. Use the combination of factors method: $\frac{\pm 1,2,4,5,10,20}{\pm 1,2,3,4,6,12}$. The only one that is not a possible factor is D, 7.
4. D. $\frac{f(m+3)-f(m)}{m+3-m} = \frac{(m+3)^2 - (m+3) + 2 - (m^2 - m + 2)}{3}$ = 2m + 2.
5. C. The maximum height of the ball is achieved at $x = \frac{-b}{2a} = \frac{-16}{-6} = \frac{8}{3}$. Thus the maximum height achieved by the ball will be at $-3(8/.)^2 + 16(8/3) + 6 = 82/3$.
6. A. Find the slope of the linear function as $\frac{5-7}{-2-2} = -0.5$. Thus y-7=(-0.5)(x-2). Q(x)=0 implies y = 0. Thus -7=(-0.5)(x-2), solve to find x = -12.
7. D. *h(1.1) + h(π) – h(-2.1) + h(0) = (1) + 3 – (-3)+0* = 7.
8. A. The critical values of f(x) are $\sqrt{11}-1$ and $-\sqrt{11}-1$. Since 0 lies between them, plug zero into the equation and test, f(0) = <5. Thus the outer interval is correct. Thus A.
9. B. $x = \frac{5}{2-3y} \rightarrow 2-3y = \frac{5}{x} \rightarrow -3y = \frac{5}{x} - 2 \rightarrow y = \left(\frac{5}{x}-2\right) \Big/ -3$ Thus $\frac{2x-5}{3x}$
10. C. The shifts given require that the domain shift to the right 3, to [3,4] and the range shift up to [3,9].
11. B. The temperature change is spread over 6 hours, thus if it is linear, $\frac{25.7}{6} = 4.2833$ degrees must change every hour. Since 3pm is 5 hours into the interval, the temperature is 5(4.28333) = 21.4166
12. B. If the shuttle can enter the atmosphere between 19 and 24,000 meters per second, then S(t) must be equal to 19000 and 24000 at those times. Thus $19000 = e^{6t} - 1$ and $24000 = e^{6t} - 1$. Solving yields *t*'s of 1.680975 and 1.6420411 minutes. Converted to seconds, this yields a window of 2.33 seconds.
13. C. Solve to find that x < 6.4375. Thus 0,1,2,3,4,5,6 are solutions, for a total of 7.
14. B. Since $ln(e^m) = m$ and $e^{\ln(m)} = m$ (*e* and *ln* cancel each other out) the equation reduces to x + 13 = 5x – 1.
15. B. f(x) = (x-3)(x+7). This becomes $x^2 + 4x - 21$. Thus k = -21.
16. C. Factor the equation x(x+7)(x-3)(x+2)(x-1) = 0. Thus the roots are -7 + 3 + -2 + 1 + 0 = -5.
17. A. Solve for the true minimum to be $\frac{10}{8} = 1.25$ Thus the closest integer is 1. f(1) = 4-10+3 = -3.
18. A. v(-1) = 2(-1)(-1) = 2. $w(2) = 2^{-2} = 0.25$. u(0.25) = 0.5 – 1 = -0.5.
19. A. The equation expands to $8a^6 - 36a^4b^4 + 54a^2b^8 - 27b^{12}$ so 8 -36 +54 -27 = -1
20. A. The function factors to $\frac{(x+3)(x-5)(x-1)}{(x+1)(x-3)(x+5)}$ so the vertical asymptotes were at (-1), (3), (-5) with a horizontal asymptote of 1. Thus 1, 3.

21. C. The max height occurs at $\dfrac{-15}{-2/3}$ 22.5 seconds.

22. B. The function lying quadrant four means the inverse reflects over the y = x to quadrant 2.

23. D. $xy = -16$ and x+y = -6. Solve to find $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{3}{8}$

24. C. Observing a pattern, it reduces to $4.5 = \sqrt{x + 2 + 4.5}$. Thus solve for x = 13.75

25. C. The recursion formula yields $p_{1999} = 90$, $p_{2000} = 100$, $p_{2001} = 123$, $p_{2002} = 143.8$, $p_{2003} = 172.38$, $p_{2004} = 204.09$.

26. C. Start with $A_c = \pi r^2$. The diameter serves as the diagonal of the square, $2\sqrt{\dfrac{A_c}{\pi}} = d$. Then use 45-45-90 triangle rules to find that the side length of the square is $\dfrac{2\sqrt{\dfrac{A_c}{\pi}}}{\sqrt{2}} = s$ Thus after reducing, the function becomes $A_s = \dfrac{2A_c}{\pi}$

27. C. R^{-1} is a function since when the coordinates are switched (e.g. x,y → y,x) none of the input values are mapped to different output values.

28. D. Plug in 3 into the M function and get $M(3) = 3^6 + 2(3^5) - k(9) + 2 - k(3)$. Solve to find k = 5. Then plug k into $x^6 + 2x^5 - 5x^2 + 2 - 5x$. Plug in (-1) to get f(-1) = 1.

29. C. $H(x) = x^{32^{0.25}} = x^8$. For all real inputs the function can take on values of zero to infinity.

30. D. For x > 0, H(x) becomes $3x^2 - 16x + 4$, for x < 0 it becomes $-3x^2 + 16x - 4$. The maximum value over that interval occurs at 0, and thus x = 5.