Gemini Solutions FAMAT State 2004

1. Only I is equivalent to 1. $\boxed{A}$

2. Move the 1 over from the other side, and for an odd-powered equation, we have
$-\frac{\text{constant}}{\text{lead coefficient}}=-\frac{12}{3}=-4$ $\boxed{A}$

3. Both positive, but different coefficients on the squared terms is an ellipse. $\boxed{C}$

4. $f(-x)=-f(x)$, and if f is defined for all reals, the only place where this identity is true is at zero. $\boxed{A}$

5. The range is part of the domain of tangent. $\boxed{A}$

6. $(\sin x+\cos y)^2-2(\sin x\cos y)=\sin^2 x+\cos^2 y=0.530001$ $\boxed{D}$

7. $a_{10}=a_1+9d=41,\ S=\frac{19}{2}(a_1+a_{19})=\frac{19}{2}(a_1+a_1+18d)=19(a_1+9d)=19\bullet 41=779$ $\boxed{B}$

8. $r=\frac{A}{s},\ \frac{\sqrt{13(7)(5)(1)}}{13}\approx 1.6$ $\boxed{B}$

9. I, IV, and VI can change, others are constants. $\boxed{D}$

10. $4^{2004}\bullet 5^{4009}=2^{4008}\bullet 5^{4008}\bullet 5=5\bullet 10^{4008}$, so it will be 500000… sum is 5 $\boxed{C}$

11. We do this like binomial expansions too: $\frac{7!}{2!2!3!}(2x)^2(3y)^2(-5z)^3=-945{,}000$ $\boxed{B}$

12. length = $\sqrt{9+16+4}ft=\frac{5.385...}{3}=1.80\,yd$ $\boxed{B}$

13. 2(12 – 12) – 4(32 – 16) + 2(48 – 24) = -16 $\boxed{B}$

14. The dihedral angle is the angle between the two planes. So we use their normal vectors, and take the dot product: $6-15-14=\sqrt{4+1+49}\sqrt{9+225+4}\cos\theta$, so $\theta=101.7...$, but since we want the *acute* angle, we subtract this from 180: 78.29° $\boxed{B}$

15. The equilateral triangle contributes a length of $4\sqrt{3}$. Thus the length of the rectangle is $18-4-4\sqrt{3}=14\sqrt{3}$. The area is $\frac{\sqrt{3}}{4}(8)^2+8\left(14-4\sqrt{3}\right)+\frac{1}{2}\pi\bullet 16=112+8\pi-16\sqrt{3}$ $\boxed{D}$

16. The distance between the center and (-1, 1) and between the center and (3, 5) is the same because the circle is tangent to the line, so both these distances are the length of the radius: $\sqrt{(h+1)^2+(k-1)^2}=\sqrt{(h-3)^2+(k-5)^2}$, which gives $h+k-4=0$. The center has to lie on the perpendicular line to the given one, because of tangency. So the line perpendicular to the one given that passes through (-1, 1) is 3y = x + 4, or 3k = h + 4. So we use these two equations to get 3k – 4 + k – 4 = 0, so k = 2, and h = 2. So $\frac{\ln 2}{2}=0.35$ $\boxed{D}$

17. Take the sum and subtract the numbers that are multiples of 2 and 3:
$$\sum_{x=1}^{2004}x-\sum_{x=1}^{1001}2x-\sum_{x=1}^{667}3x=\frac{2004}{2}(1+2004)-2\bullet\frac{1001}{2}(1+1001)-3\bullet\frac{667}{2}(1+667)=337{,}674\ \boxed{A}$$

18. We can take ${}_{16}C_3$, but we must subtract the degenerate triangles made up of straight lines. We subtract $10\left({}_4C_3\right)$ because of the 4 horizontal, 4 vertical, and 2 diagonal lines that contain 4 that

we could choose 3 at a time from. We subtract $4({}_3C_3)$ because of the 4 ways to choose the 3 points right in a row. This leaves us with 516 $\boxed{D}$

19. Obviously a root is $4+7i$, since imaginary roots appear in conjugate pairs. Since the sum of the roots is zero (because there is no quadratic term), the other root must be -8. So our cubic is $(x-4-7i)(x-4+7i)(x+8)=0$, which you can expand or keep using the same technique taking the sum of the roots taken two at a time, and the product of the roots. Either way, it becomes $y=x^3+x+65\bullet 8$. $\frac{(65\bullet 8)^1}{65}=8$ $\boxed{C}$

20. We can have $x^2-5x+5=1$, which gives x = 1 and x = 4. Or we can have $x^2-9x+20=0$, which gives 4 and 5. Notice we don't get undefined forms when we plug these back in, so our answers check, and their sum is 10. $\boxed{D}$

21. $7^{2004}\left(1-2\bullet 7+7^2-7^3\right)=n\bullet 7^{2004}$, $n=-307$ $\boxed{A}$

22. You cannot add infinite geometric series with ratios bigger than one. Sum is infinity $\boxed{E}$

23. We can factor 4,172,004 into $2^2\bullet 3^2\bullet 17^2\bullet 401$, and we add one to each exponent and multiply: $3\bullet 3\bullet 3\bullet 2=54$ $\boxed{C}$

24. Think of x, y, and z as roots of a cubic polynomial. The sum is -1, the sum taken two at a time is -17, and the product of the roots is -15. Thus if we have a cubic of the form $y=x^3+bx^2+cx+d$, b = 1, c = -17, and d = 15 yields $y=x^3+x^2-17x+15$. 1 is a root, so synthetically divide and factor to find the others. So x = -5, y = 1, and z = 3. $\frac{x}{yz}=-\frac{5}{3}$ $\boxed{B}$

25. First term: $b^{-\frac{1}{2}}$. Second term: $\left(-\frac{1}{2}\right)b^{-\frac{3}{2}}(-2)$. Third term: $\frac{3}{8}(b)^{-\frac{5}{2}}(-2)^2$. Fourth term: $-\frac{5}{16}(b)^{-\frac{7}{2}}(-2)^3$, so our coefficient is 2.5 $\boxed{C}$

26. Set $\ln x=2$, so $x=e^2$, so plug in that everywhere there is x: $\left(e^2\right)^4+5\left(e^2\right)^3-2\left(e^2\right)^2+3\left(e^2\right)-7\approx 4904$ $\boxed{D}$

27. The radius of the circle is $\frac{5.7}{2}$, so we want the area of the square minus the area of the circle divided by the area of the ellipse: $\frac{5.7^2-\pi\left(\frac{5.7}{2}\right)^2}{\left(\frac{19}{2}\right)\left(\frac{16.4}{2}\right)\pi}=0.02849...\Rightarrow 0.03$ $\boxed{C}$

28. We use $A = P\left(1+\frac{r}{n}\right)^{nt}$, where P is the principle, r is the rate, n is the number of times compounded yearly, and t is time. Here, $1125\left(1+\frac{\frac{\pi}{100}}{2}\right)^{2t} = 2222e$, and after taking the natural log of both sides, we have $2t\ln(1.0157...) = 1.68...$ which yields $t \approx 54\text{ years}$ $\boxed{B}$

29. $c = 4\sqrt{2},\ e = 8$, so $\frac{4\sqrt{2}}{a} = 8,\ a = \frac{1}{\sqrt{2}}$ $c^2 = a^2 + b^2$, so $32 = \frac{1}{2} + b^2,\ b = \sqrt{\frac{63}{2}}$. Since this is the hypotenuse of a right triangle going into the second and fourth quadrant. Divide by the square root of 2, and we have the x- and y-coordinates: $\frac{\sqrt{63}}{2}$ $\left(-\frac{3\sqrt{7}}{2}, \frac{3\sqrt{7}}{2}\right)$ $\boxed{C}$

30. $2 \bullet 4 \bullet \pi = 8\pi$ $\boxed{B}$