INTEGRATION TEST - CALCULUS FAMAT STATE CONVENTION 2004

1. $\int_1^2 \sqrt{\frac{2}{x^2}}\, dx =$

a) $\sqrt{2}\ln(2)$ b) $\frac{\sqrt{2}}{2}$ c) $\frac{7\sqrt{2}}{4}$ d) $\frac{3\sqrt{2}}{2}$ e) NOTA

2. Find the area enclosed by the graphs of $y = x^2$ and $y = 2x + 3$.

a) $\frac{16}{3}$ b) $\frac{32}{3}$ c) $\frac{38}{3}$ d) $\frac{40}{3}$ e) NOTA

3. If $\int_a^b f(x)dx = 0$, then which of the following must be true?

a) f(x) = 0 b) a = b c) f(-x) = -f(x) d) At least one of the choices a, b, or c. e) NOTA

4. $\int \frac{x + e^x}{xe^x} dx =$

a) $-e^{-x} - \frac{1}{x^2} + C$ b) $e^{-x} - \ln|x| + C$ c) $e^{-x} + \ln|x| + C$ d) $-e^{-x} + \ln|x| + C$ e) NOTA

5. Find the volume of a solid given that its base is an isosceles right triangle with legs of length 4 and cross sections perpendicular to one of its legs are semicircles.

a) $\frac{\pi}{2}$ b) π c) $\frac{8\pi}{3}$ d) $\frac{16\pi}{3}$ e) NOTA

6. $\int \frac{\sec\sqrt{t}}{\sqrt{t}} dt =$

a) $\ln\sqrt{\sec\sqrt{t}} + \ln\sqrt{\tan\sqrt{t}} + C$ b) $\frac{\ln\left|\sec\sqrt{t} + \tan\sqrt{t}\right|}{2} + C$

c) $\ln\sqrt{\sec\sqrt{t} - \tan\sqrt{t}} + C$ d) $2\ln\left|\sec\sqrt{t} + \tan\sqrt{t}\right| + C$ e) NOTA

7. If $u = \sqrt{4x+1}$, then $\int_0^2 3x\sqrt{4x+1}\,dx =$

a) $\frac{3}{8}\int_1^3 (u^4 - u^2)du$ b) $\frac{3}{4}\int_1^3 (u^3 - u)du$ c) $\frac{3}{4}\int_0^2 (u^4 - u^2)du$ d) $\frac{3}{8}\int_1^3 (u^3 - u)du$ e) NOTA

8. $y^2 x\dfrac{dy}{dx} - x + 1 = 0$. If y(1) = 3, find the tenths digit of y(2).

a) 0 b) 2 c) 4 d) 6 e) NOTA

9. The region bounded by y = sec(x), y = 0, x = 0, and x = $\dfrac{\pi}{4}$ is rotated about the x-axis. Find the volume of the resulting solid.

a) $\dfrac{\sqrt{2}\pi}{2}$ b) $\dfrac{\pi}{2}$ c) π d) $\dfrac{\pi^2}{4}$ e) NOTA

10. Which of the following statements are true?

i) $\int_0^{\pi/4} \sin(x)dx \le \int_0^{\frac{\pi}{4}} \cos(x)dx$

ii) $\int_{-1}^{1} |x| dx = 0$

iii) $\int_1^3 f(t)dt + \int_5^3 f(t)dt = \int_5^1 f(t)dt$

iv) $\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} (\cos(x) - x^2)dx > 0$

a) i, iv b) ii, iv c) i, iii, iv d) i, ii, iii, iv e) NOTA

11. $\lim_{n\to\infty} \dfrac{e^{\frac{2}{n}} + e^{\frac{4}{n}} + e^{\frac{6}{n}} + \cdots + e^{\frac{2n}{n}}}{n} =$

a) 0 b) $\dfrac{e^2}{2}$ c) $\dfrac{2e^2 - 1}{2}$ d) $\dfrac{e^2 - 1}{2}$ e) NOTA

12. Find the number(s) b such that the average value of f(x) = 2 + 6x – 3x^2 on [0,b] is equal to 3.

a) $\dfrac{3 \pm \sqrt{33}}{6}$ b) $\dfrac{3747}{1000}$ c) $\dfrac{3 \pm 2\sqrt{3}}{3}$ d) $\dfrac{3 \pm \sqrt{13}}{2}$ e) NOTA

13. Let R be the region bounded by the curves y = tan(x^2), x = 1 and y = 0. Use the Midpoint Rectangular Approximation Method with n = 4 to estimate the volume by rotating R about the x-axis. Round your answer to the nearest thousandth.

a) 0.278 b) 0.437 c) 0.875 d) 1.139 e) NOTA

14. If $x\sin(\pi x) = \int_0^{x^2} f(t)\,dt$, where f is a continuous function, find f(4).

a) $\frac{\pi}{2}$ b) $\frac{3\pi}{4}$ c) π d) 2π e) NOTA

15. A student forgot the Product Rule for differentiation and made the mistake of thinking that $(fg)' = f'g'$. However he was lucky and got the correct answer. The function f that he used was $f(x) = e^{x^2}$ and the domain of his problem was (0.5, ∞). Find $g(x)$.

a) $\frac{Ce^x}{2\sqrt{2x-1}}$ b) $Ce^x\sqrt{2x-1}$ c) $e^x \ln(2x-1) + C$ d) $Ce^x \ln(2x-1)$ e) NOTA

16. The value of a yacht in dollars after t years of use is $V(t) = 300{,}000(2^{-0.15t})$. To the nearest dollar what is the average value of the yacht over its first 10 years?

a) \$126,036 b) \$137,048 c) \$197,718 d) \$285,247 e) NOTA

17. Let $R = \int_{\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt[4]{2}}} \frac{2x}{\sqrt{1-x^4}}\,dx$. Which of the given intervals contains R?

a) $(-\infty, 0.1]$ b) $(0.1, 0.2]$ c) $(0.2, 0.3]$ d) $(0.3, 0.4]$ e) NOTA

18. The rate of change in volume V of a melting spherical snowball is proportional to the surface of the area of the snowball, $\frac{dV}{dt} = -kS$. If the length of the radius of the ball at t = 0 is r = 3 and at t = 5, r = 1, what is the length of the radius at t = 7?

a) 0.2 b) 0.4 c) 0.5 d) 0.75 e) NOTA

19. A particle moves along a line so that any time $t \geq 0$, its velocity is given by $v(t) = \frac{t^2}{2+t^3}$. What is the total distance traveled by the particle from t = 1 to t = 5?

a) $\ln\sqrt[3]{124}$ b) $3\ln(\frac{127}{3})$ c) $\frac{\ln(5)}{3}$ d) $\frac{\ln(127) - \ln(3)}{3}$ e) NOTA

20. The area in the first quadrant of the region bounded by $f(x) = x$ and $g(x) = ax^2$, $a > 0$, is equal to 0.8. Find the tenths digit of $\ln(a)$.

a) 2 b) 5 c) 7 d) 8 e) NOTA

21. The acceleration at time $t > 0$ of a particle moving along the x-axis is $a(t) = \sin(2t)$. If at $t = \frac{\pi}{4}$, the velocity is 3 and the position is $x = 6$, then at $t = \frac{\pi}{2}$ the position $x(t) = ?$

a) $\frac{32-\pi}{8}$ b) $\frac{25+3\pi}{4}$ c) $6+\frac{3\pi}{4}$ d) $\frac{2\pi+6}{3}$ e) NOTA

22. The region in the first quadrant bounded by $y = \frac{1}{4+x^2}$, the coordinate axes and $x = k$ is rotated about the y-axis. If the resulting solid has an area of $\pi\ln(2)$, find the tenths digit of $\ln(k)$.

a) 1 b) 2 c) 6 d) 9 e) NOTA

23. $\int_0^a (2x+\sqrt{a^2-x^2})dx, a>0 =$

a) $\frac{a^2(4+\pi)}{4}$ b) $\frac{a^2(2+\pi)}{2}$ c) $\frac{2a^2(1+\pi)}{3}$ d) $\frac{a^2(1+\pi)}{2}$ e) NOTA

24. The value of the integral $\int_a^b (14-31x-10x^2)dx$ reaches its maximum value on $[a,b]$. Find the tenths digit of $\ln|ab|$.

a) 1 b) 3 c) 6 d) 8 e) NOTA

25. If $\int_a^b f(x)dx = 3$, $\int_a^b g(x)dx = 7$ and $\int_b^c g(x)dx = 10$, which of the following integrals can be evaluated based on the given information? i) $\int_b^a f(x)dx$ ii) $\int_a^c g(x)dx$

iii) $\int_a^c (f(x)+g(x))dx$ iv) $\int_a^b f(x)g(x)dx$ v) $\int_a^b (2f(x)-3g(x))dx$ vi) $\int_a^b \frac{f(x)}{g(x)}dx$

a) i,ii,iii,v b) i,ii,iv,v c) i,ii,v d) i,ii,iii,iv,v,vi e) NOTA

26. AM radio signals are transmitted by sending a high frequency wave whose amplitude varies in the pattern of the sound being carried. f(x) = 4sin(x)sin(8x), where f(x) is the strength of the signal at any instant x. Which below could be used to find $\int_0^{\frac{\pi}{2}} f(x)dx$, if $u = 9x$ and $v = -7x$?

a) $\int_0^{\frac{\pi}{2}} (\cos(-7x) - \cos(9x))dx$ b) $\frac{-2}{9}\int_0^{\frac{\pi}{2}} \cos(u)du + \frac{2}{7}\int_0^{\frac{\pi}{2}} \cos(v)dv$

c) $\frac{-2}{9}\int_0^{\frac{9\pi}{2}} \cos(u)du - \frac{2}{7}\int_0^{\frac{7\pi}{2}} \cos(v)dv$ d) $2[\frac{-1}{9}\int_0^{\frac{9\pi}{2}} \cos(u)du + \frac{1}{7}\int_{\frac{-7\pi}{2}}^{0} \cos(v)dv$ e) NOTA

27. $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cot(x)dx =$

a) $\ln\left(\frac{\sqrt{2}}{2}\right)$ b) $\frac{\ln(2)}{2}$ c) $\ln(2)$ d) $2\ln(2)$ e) NOTA

28. $\int (\sin(x) + \cos(x))^2 dx =$

a) $\frac{x - \cos(2x)}{2} + C$ b) $\frac{2x - \cos(2x)}{2} + C$ c) $\frac{x + \cos(2x)}{2} + C$

d) $\frac{2x + \cos(2x)}{2} + C$ e)NOTA

29. If the substitution $\sqrt{x} = \sin(y)$ is made in $\int_0^{\frac{1}{2}} \frac{\sqrt{x}}{\sqrt{1-x}} dx$, then the resulting integral is

a) $\int_0^{\frac{1}{2}} \sin^2(y)dy$ b) $2\int_0^{\frac{\pi}{6}} \sin^2(y)dy$ c) $\int_0^{\frac{\pi}{4}} \tan(y)\sin(y)dy$ d) $2\int_0^{\frac{\pi}{4}} \sin^2(y)dy$ e) NOTA

30. $f(x) = x^n + 1$, where n is an odd natural number. Find $\int_0^2 f^{-1}(x)dx$.

a) 0 b) $\frac{n}{n+1}$ c) $\frac{2n}{n+1}$ d) $\frac{-2n}{n+1}$ e) NOTA