

Question # 1
Alpha Bowl 2004

Let $f(x) = 128x^{15} - 2x^{13} - 2x^{10} + 3x^8 - x^6 - x^5 + x^3 - 5x^2 + 120$

Find the value of (A + B + C + D + E + F)

- A) The sum of the possible rational roots of $f(x)$.
- B) According to DesCarte's Rules of Signs, what is the highest number of possible positive real roots of $f(x)$.
- C) According to DesCarte's Rules of Signs, Find the least number of possible negative real roots of $f(x)$.
- D) (True or False) True = 1, False = 2: There must be at least one positive real root.
- E) The sum of the roots of $f(x)$.
- F) The product of the roots of $f(x)$.

Question # 2
Alpha Bowl 2004

Given below are four different triangles, find (A + B + C + D), round this final sum to the nearest thousandths place. Note: Capital letters represent angles and lower case letters represent sides opposite the angle.

- A) In $\triangle PET$, $p = 5$, $e = 12$, $E = 90$ degrees, find the area of this triangle.
- B) In $\triangle RON$, $R = 79$ degrees, $o = 9$, $r = 12$, find the area of this triangle.
- C) In $\triangle FIG$, $f = 11$, $i = 15$, $g = 18$, find the area of this triangle.
- D) Find the area of the triangle formed by the points (2, 5), (-9, 3), and (4, 2004).

Question # 3
Alpha Bowl 2004

Find the solution, (which is a four letter word), by using the following set of codes { 1=a, 2=b, 3=c, 4=d, ..., 25= y, 26=z }, and solving these four cases: The answer formed uses the answer for case 1 as its first letter, case 2 as its second letter and so on.

case 1: Evaluate: $\sin\left(\frac{\pi}{2}\right) + \log_5 125 + \ln e^{15} - \log_2 8 + \log_x x^3$.

case 2: The smallest factor, other than 1, for 1,972,883.

case 3: One more than the denominator of the simplified form, expressed as one term, for the exact value of $\cos(105^\circ)$.

case 4: Four times the radius of this circle: $x^2 + y^2 - 4x + 6y - 23 = 0$.

Question # 4
Alpha Bowl 2004

Evaluate the following expression to the nearest thousandths place:

$$\frac{\sin 5^\circ \sin 10^\circ \sin 15^\circ \sin 20^\circ \sin 30^\circ \sin 45^\circ \sin 60^\circ \sin 70^\circ \sin 75^\circ \sin 80^\circ \sin 85^\circ}{\cos 5^\circ \cos 10^\circ \cos 15^\circ \cos 20^\circ \cos 30^\circ \cos 45^\circ \cos 60^\circ \cos 70^\circ \cos 75^\circ \cos 80^\circ \cos 85^\circ}$$

Question # 5
Alpha Bowl 2004

In parallelogram ABCD, AB=16, BC=20, and the angle bisector of angle A splits the angle so that each half is 60 degrees. Find the area of a similar parallelogram EFGH if FG=27.

Question # 6
Alpha Bowl 2004

Let x be the coefficient of the 2004^{th} term of $(2004 - y)^{2004}$: Find $2 \sum_{k=1}^{20} (k^2 + x)$

Question # 7
Alpha Bowl 2004

List all of the choices which are always true statements.

- F) For a polynomial function with an even degree: Its graph will have an even amount of solutions that are both positive and negative.
- M) For a polynomial function with an odd degree: Its graph must cross the x-axis exactly one time.
- A) For a polynomial function with an odd degree: Its graph must cross the x-axis less than one time.
- R) For a polynomial function with an even degree: Its graph will not have two y-intercept(s).
- L) For a polynomial function with odd degree: Its graph must cross the x-axis at least one time.
- I) For a polynomial function with an even degree: Its graph is symmetric to the y-axis.
- N) For a polynomial function with odd degree: Its graph is symmetric to the x-axis.
- S) For a polynomial function with odd degree: Its end behavior, as x approaches infinity and negative infinity, is the same.

Question # 8
Alpha Bowl 2004

Find the area of the conic section with the following known information: This conic section has Two foci points of $(-2, 5)$, $(-2, 11)$, the eccentricity is between zero and one, and in addition this conic section has a minor axis with length of 12.

Question # 9
Alpha Bowl 2004

Find $A + B + C + D$ for the following:

A) Find the $\lim_{x \rightarrow 4} \left(\frac{x^2 + 2x - 24}{x - 4} \right)$

B) The limit of $\lim_{x \rightarrow 3} (f(x)) = 8$, find $\lim_{x \rightarrow 3} (5f(x))$.

C) Find the $\lim_{x \rightarrow \pi} (f(x))$, if $f(x) = 12$.

D) Find the $\lim_{x \rightarrow -3} \left(\frac{x^3 + 27}{x + 3} \right)$

Question # 10
Alpha Bowl 2004

Let vector R represent the cross product of $(1, -6, 5)$ and $(3, -2, 1)$. Let L equal the inner product of vector R and the vector represented as a sum of unit vectors of $-6i + 3j - k$. Let vector A equal RL . Find the magnitude of this new vector A .

Question # 11
Alpha Bowl 2004

Find $A + B + C$ for the following:

A) Find the 2004th term in this sequence: $-3, -12, -21, -30 \dots$

B) How many terms are in a sequence with the first term of 64 and the last term 888 and common difference is 4 .

C) Alpha and Precal went up a hill to fetch a pail of water. Alpha kept filling the bucket, then dumping it into Precal's larger bucket, with less each time until the end of time (which never came). How many total pints of water did Alpha pour into Precal's bucket, if Alpha's pattern went as follows, starting with the first bucket of 90 pints, second bucket of 30

pints, third bucket of 10 pints, fourth bucket of $\frac{10}{3}$ pints, etc...

Question # 12
Alpha Bowl 2004

Use the following matrices: For parts I, II, III, IV, V, and VI: If an answer is a matrix then use only the entry found in the second row and second column, if an answer is a real number then use the real number found. Find the sum of I + II + III + IV + V + VI.

$$\mathbf{A} = \begin{bmatrix} 2 & -4 \\ -1 & -2 \\ 3 & -8 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} -5 & 7 \\ -4 & -8 \\ 1 & 4 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 2 & 6 \\ -4 & 8 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 0 \\ 2 & 2 & -1 \end{bmatrix}, \mathbf{E} = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & -2 \\ -1 & -1 & 1 & 2 \\ 2 & 0 & 0 & 4 \end{bmatrix}$$

- I) Find $A - B$
 II) Find BC
 III) Find the value of C^3
 IV) Find the determinant of E
 V) Find $2004B - 2004A$
 VI) Find the determinant of D^2

Question # 13
Alpha Bowl 2004

Write the polynomial of least degree with the following roots: ± 4 , $2i$, and $2 - 3i$.

Question # 14
Alpha Bowl 2004

Let $9^{x+1} = 8^{2x}$. The eccentricity of a conic section is $\frac{13}{x}$. The distance between foci points of this conic section is 52. Find the length of the conjugate axis rounded to the thousandths place.

Question # 15
Alpha Bowl 2004

All answers for M, A, T, and H can be expressed as rational numbers. Find the sum of the Numerators for the simplified fractions of questions M, A, T, and H.

M) Evaluate: $\frac{2003!(5!)}{2004!}$

A) Find the number of distinct permutations of 11 coins if three are identical pennies, six are identical quarters, and two are identical dimes.

T) Find the probability of getting two green tokens and a three red tokens, in this order, from Five consecutive draws out of a bag containing 9 red, 8 blue and 6 green tokens.
(The tokens are not replaced after each draw).

H) What is the probability of getting tails 8 times in a row when tossing a fair coin which has a head on one side and a tail on the other (the coin is picked up and tossed again each time) ?

1) $\frac{129}{16}$ or 8.0625

2) 11145.547

3) SKEW

4) 2.303

5) $\frac{1458\sqrt{3}}{5}$

6) -160,634,900

7) R, L, and I

8) $18\pi\sqrt{5}$

9) 89

10) $12\sqrt{13}$

11) -17,688

12) -11994

13)

$x^6 - 4x^5 + x^4 + 48x^3 - 220x^2 + 256x - 1$

14) 51.807

15) 4649