

POLYNOMIAL AND RATIONAL FUNCTIONS - THETA
2003 Mu Alpha Theta National Convention

For all questions, answer E. "NOTA" means none of the above answers is correct.

1. Find the value of k so that the two given lines will be perpendicular to each other.

$$3x - 4ky = 17$$

$$2x + 9y = -22$$

- A. $\frac{1}{6}$ B. $\frac{2}{3}$ C. $-\frac{1}{6}$ D. 6 E. NOTA

2. Find the slope of the linear line that passes through the points $(k-4, -3), (-k, 2), (5, 11)$.

- A. $\frac{11}{23}$ B. $\frac{23}{14}$ C. $\frac{1}{322}$ D. Undefined E. NOTA

3. A wooden rectangular box has an outer surface area of 4400 cm^2 with a square base and a height of 45 cm. Find the volume of the box to the nearest cubic centimeter.

- A. 2212 cm^3 B. 18000 cm^3 C. 72000 cm^3 D. 544500 cm^3 E. NOTA

4. Which of the following statement(s) is(are) **FALSE**?

I. The discriminant for the function $f(x) = 3x^2 + 9x + 10$ is $\sqrt{39}$.

II. The function $q(x) = 2x^2 + 10x - 13$ has two imaginary complex conjugate roots.

III. The function $g(x) = x^2 - 13$ has two distinct roots.

- A. I only B. II only C. III only D. I, II only E. NOTA

5. Given that one of the roots for the quadratic equation $Ax^2 + Bx + C = 0$ is

$$\frac{5}{8} + 2i\sqrt{5}, \quad A = 1, \text{ and } A, B, C \text{ are relative prime; find } \frac{A+C}{B}.$$

- A. $-\frac{16}{1305}$ B. $-\frac{16}{145}$ C. $-\frac{1241}{80}$ D. $-\frac{1369}{80}$ E. NOTA

6. Find $A + B$.

Let $A =$ sum of the roots for the polynomial $9x^4 + 3x^2 - 2x + 4 = 0$.

Let $B =$ product of the roots for the polynomial $16x^5 - 3x^4 - x^3 + 2x^2 + x - 9 = 0$.

- A. $\frac{11}{48}$ B. 6 C. $\frac{9}{16}$ D. $\frac{43}{48}$ E. NOTA

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7. Which of the following statement(s) is(are) **TRUE**?

- I. The axis of symmetry of the function $q(x) = \frac{1}{2}x^2 - 2x + 5$ is the line $y = 2$.
- II. The vertex of the function $f(x) = \frac{3}{4}x^2 - 2x + 3$ is $\left(\frac{4}{3}, 0\right)$.
- III. The maximum value on the graph of $g(x) = -\frac{1}{3}x^2 + 6x$ is 27.

A. I only B. I, II only C. III only D. I, III only E. NOTA

8. A certain video game at an arcade has 360 users on a daily basis, each of whom pay \$0.50 to play. The arcade owners have determined that for every \$0.05 increase in price to play the game, 20 fewer people will play. What should the arcade charge people to play the video game in order to collect \$196.00 in one day?

A. \$0.10 B. \$0.20 C. \$0.60 D. \$0.70 E. NOTA

9. Patty needs to build a hollow metal sphere with a wall that is 4.5 cm thick. If the total volume of metal available is $425\pi \text{ cm}^3$, what is the radius of the sphere to the nearest tenth?

A. 3.4 cm B. 7.9 cm C. 9.4 cm D. 27.4 cm E. NOTA

10. Solve the following inequality over \mathfrak{R} : $x^2 - 12x + 32 \geq -3$.

- A. $\{x : x > 7 \text{ or } x < 5\}$ B. $\{x : x \leq 7\}$
C. $\{x : x \geq 5\}$ D. $\{x : x \geq 7 \text{ or } x \leq 5\}$ E. NOTA

11. Solve the following inequality over \mathfrak{R} in terms of n : $x^{4n} + 3x^{2n} - 4 < 0$.

A. $\{\}$ B. $\{x : -1 < x < 1\}$ C. $\{x : x > 2\}$ D. $\{x : x < 2\}$ E. NOTA

12. Which of the following statement(s) is(are) **TRUE**?

- I. A polynomial of degree n can have at most $n+1$ turning points.
- II. In a graph of a polynomial function of even degree, both ends go up or both ends go down.
- III. Given a rational function, $f(x) = \frac{p(x)}{q(x)}$, if the degree of polynomial $p(x)$ is less than the degree of $q(x)$, then there is no horizontal asymptote.

A. I only B. II only C. I, III only D. I, II only E. NOTA

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13. Let $f(x) = 3x + 2$ and $g(x) = 2x^2 - 3$. Evaluate $(g + f)(-3)$.

- A. -120 B. 0 C. 8 D. 26 E. NOTA

14. Express in simplest terms.

$$\frac{\left(u - v + \frac{2v^2}{u + v}\right)\left(\frac{1}{u^2} - \frac{1}{v^2}\right)}{\left(u^3 - \frac{u^3v + v^4}{u + v}\right)}$$

- A. $\frac{(u^2 + v^2)(v^2 - u^2)}{(u^2v^2)(u^4 + v^4)}$ B. u^2v^2
C. $-\frac{1}{u^2v^2}$ D. $u^2 + v^2$ E. NOTA

15. In a certain animal population, a birth occurs every 24 minutes and a death every half hour. An individual moves out of the region every 1.5 hours, and a new individual moves into the region every 4.5 hours. How many hours does it take, on the average, for the population to increase by 1 individual?

- A. $\frac{10}{29}$ hours B. $\frac{18}{73}$ hours C. 18 hours D. $\frac{1}{18}$ hours E. NOTA

16. How many grams, to the nearest hundredth, of sodium chloride must be added to 451.2 g of a 28.5% sodium chloride solution to produce a 46.4% sodium chloride solution?

- A. 112.96g B. 150.68g C. 297.19g D. 630.50g E. NOTA

17. Which of the following must be **TRUE** for the function $f(x) = x|x|$?

- I. The function is even.
II. The graph of the function is symmetric with respect to the origin.
III. The function is neither even nor odd.
A. I only B. I, II only C. II only D. III only E. NOTA

18. Consider the equation $ky + 4x = 9$. For what value of k will the slope be 2?

- A. -2 B. -1 C. $\frac{1}{2}$ D. 2 E. NOTA

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19. The square of the period of revolution of an artificial satellite varies directly as the cube of its distance from the center of Earth. A satellite 25,000 km from the center of Earth has a period of revolution of 7.25 hours. What would be the period of revolution, to the nearest tenth, of a satellite 32,485 km from the center of Earth?

- A. 5.1 B. 10.7 C. 25.6 D. 123.4 E. NOTA

20. Which of the following statement(s) is(are) **FALSE**?

- I. For any positive integers, a, b, c , and d , $\frac{a}{b} < \frac{c}{d}$ if and only if $ad < bc$.
II. If a fraction in lowest terms can be represented by a terminating decimal, its denominator can have no prime factors other than 5.
III. The sum of a rational number and an irrational number is always irrational.

- A. I, II only B. I, III only C. II only D. III only E. NOTA

21. Solve for the inequality over \mathcal{R} : $x^3 - 4x^2 + x + 6 \leq 0$

- A. $(-\infty, 3]$ B. $\{-1, 2, 3\}$
C. $[-1, 2] \cup [3, \infty)$ D. $(-\infty, -1] \cup [2, 3]$ E. NOTA

22. In pottery class, the assignment is to build a cylindrical container made out of $72\pi \text{ cm}^3$ of clay, whose walls and base have a thickness of 1.5 cm (it has no top). What is the outside radius, to the nearest tenth, of the container if the height and radius are equal?

- A. 4.4 B. 5.4 C. 8.1 D. 9.2 E. NOTA

23. The vertex of the parabola $y = x^2 - 8x + C$ will be a point on the x -axis if the value of C is what?

- A. -16 B. -4 C. 4 D. 16 E. NOTA

24. The polynomial equation $4x^6 - 5x^5 + 4x^3 - 3x^2 - 2x + 3 = 0$ can **NOT** have which of the following combinations of roots?

- A. 6 imaginary B. 2 positive, 2 negative, 2 imaginary
C. 6 negative D. 4 negative, 2 imaginary E. NOTA

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25. How many asymptotes does the equation $y = \frac{x^3 + 2x^2 - 3}{x^2 - 1}$ have?
A. 2 vertical, 0 horizontal B. 2 vertical, 1 horizontal
C. 1 vertical, 2 horizontal D. 1 vertical, 1 horizontal E. NOTA
26. Which of the following statement(s) is(are) **TRUE**?
I. The least positive integral upper bound for the polynomial $x^3 - x^2 + 2x + 9 = 0$ is -2 .
II. Every polynomial of odd degree with real coefficients has at least one real root.
III. If $P(x)$ is a polynomial with real coefficients, and $P(a + bi) = 0$, then $P(a - bi) = 0$, for any real numbers a and b .
A. I only B. II, III only C. III only D. I, II, III only E. NOTA
27. Solve for x : $\frac{3x}{2x - 8} = \frac{54}{9x - 36} + 3$
A. 4 B. 6 C. -4 D. $\{ \}$ E. NOTA
28. Which of the following statement(s) is(are) **FALSE**?
I. Over the set of complex numbers, $x - r$ is a factor of a polynomial $F(x)$ if and only if r is a root of $F(x) = 0$.
II. Synthetic substitution (or synthetic division) applies only to polynomials that have a degree n greater than 3.
III. A quadratic equation with rational coefficients has rational roots if and only if its discriminant is the square of a rational number.
A. I, III only B. I, II only C. II only D. III only E. NOTA
29. If the equation of the line perpendicular to $3x - 7y + 2 = 0$ is expressed in $Ax + By + C = 0$ form and passes through the point $(-3, 2)$, find $A - B + C$.
A. -5 B. 5 C. 19 D. 25 E. NOTA
30. Which of the following statement(s) is(are) **FALSE**?
I. A polynomial with degree 2 can have both a minimum and a maximum.
II. Every even function has symmetry about the origin.
III. If $P(x)$ is a polynomial with real coefficients, then the number of positive roots of $P(x) = 0$ is either equal to the number of variations of sign of $P(x)$ or is less than this number by a positive odd integer.
A. I, II only B. II, III only C. I, III only D. I, II, III only E. NOTA