

$$1) \text{ Area} = \int_2^5 \frac{x+3}{2} dx + \int_5^7 \frac{4x-8}{3} dx = \frac{245}{12} \quad \text{C}$$

$$2) \text{ Area} = \int_{-\infty}^0 e^x dx + \int_0^{\infty} e^{-x} dx = 2 \quad \text{D}$$

$$3) \text{ Polar Area} = \frac{1}{2} \int_{\alpha}^{\beta} r^2 d\theta = \frac{1}{2} \int_{-\frac{\pi}{8}}^{\frac{\pi}{8}} (5 \cos(4\theta))^2 d\theta = \frac{25\pi}{16} \quad \text{B}$$

$$4) \text{ Using the disc method, the volume} = \pi \int_1^e \left(\frac{1}{x}\right)^2 dx = \frac{\pi(e-1)}{e} \quad \text{A}$$

$$5) \text{ Area of solid} = \int_0^{\frac{\pi}{2}} \pi \left(\frac{\sin(2x)}{2}\right)^2 = \frac{\pi^2}{32} \quad \text{B}$$

$$6) \int_0^4 x\sqrt{1+x^4} dx \approx \frac{1}{2} \left[ 0\sqrt{1+0^4} + 2(1\sqrt{1+1^4}) + 2(2\sqrt{1+2^4}) + 2(3\sqrt{1+3^4}) + 4\sqrt{1+4^4} \right] \approx 68.889 \quad \text{D}$$

$$7) \text{ Area} = \int_1^2 \left[ (-x^2 + 4x + 5) - \left(\frac{-7x}{4} + \frac{39}{4}\right) \right] dx + \int_2^3 \left[ \left(18 - \frac{9x}{2}\right) - \left(\frac{-7x}{4} + \frac{39}{4}\right) \right] dx = \frac{35}{12} \quad \text{A}$$

8)  $x^2 + y^2 + 10x - 6y - 2 = 0$  is the circle centered at  $(-5, 3)$  with radius 6.  $\Rightarrow$  Theorem of Pappus  $\therefore$   
 Volume =  $(\pi 6^2)(2\pi)(4 - (-5)) = 648\pi^2 \quad \text{E}$

$$9) \text{ Area of surface} = \int_0^{\pi} (2 \cos \theta)(\cos \theta) \sqrt{(2 \cos \theta)^2 + \left(\frac{d}{d\theta}(2 \cos \theta)\right)^2} d\theta = 4\pi^2 \quad \text{C}$$

$$10) \text{ Volume} = 2\pi \int_0^3 x e^{2x+1} dx = \frac{e\pi(5e^6 + 1)}{2} \quad \text{B}$$

$$11) \pi \int_0^{\frac{\pi}{4}} [\pi^2 - (\pi - \tan x)^2] dx = -\pi \int_0^{\frac{\pi}{4}} (\pi - \tan x)^2 dx + \frac{\pi^4}{4} \quad \text{C}$$

$$12) \int_a^{-1} [x(x+5)] dx = \frac{77}{6} = \frac{-a^3}{3} - \frac{5a^2}{2} + \frac{13}{6} \therefore a = -8 \quad \text{E}$$

$$13) \text{ Area} = \int_0^2 y^{\frac{7}{2}} dy = \frac{32\sqrt{2}}{9} \quad \text{D}$$

$$14) \frac{2\pi}{18} \left[ -\pi \sin -\pi + 4 \frac{-2\pi}{3} \sin \frac{-2\pi}{3} + 2 \frac{-\pi}{3} \sin \frac{-\pi}{3} + 4(0)\sin(0) + 2 \frac{\pi}{3} \sin \frac{\pi}{3} + 4 \frac{2\pi}{3} \sin \frac{2\pi}{3} + \pi \sin(\pi) \right] \approx 6.33$$

A

$$15) \text{ Surface area} = 2\pi \int_a^b \sqrt{1 + (y')^2} dx \Rightarrow 2\pi \int_0^{\frac{\pi}{4}} x \sqrt{\frac{x^4 + 2x^2 + 2}{x^4 + 2x^2 + 1}} dx \quad \text{B}$$

$$16) \text{ Volume} = 2\pi \int_1^2 \frac{x-1}{x^2+1} dx = 2\pi \left( \frac{\ln(5/2)}{2} + \arctan\left(\frac{1}{2}\right) - \frac{\pi}{4} \right) \quad \text{D}$$

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$$17) \text{ Volume} = \pi \int_3^7 (-x^2 + 10x - 21)^2 dx = \frac{512\pi}{15} \quad \boxed{\text{E}}$$

$$18) x^2 + y^2 - 8x + 12y + 27 = 0 \text{ is a circle with radius 5 centered at } (4, -6). \text{ Distance from point to line} = \frac{|5(4) + -12(-6) + 77|}{\sqrt{5^2 + (-12)^2}} = 13. \text{ Using the Theorem of Pappus, the surface area} = ((2)5\pi)((2)13\pi) = 260\pi^2 \quad \boxed{\text{C}}$$

$$19) \text{ Volume} = \int_{-2}^3 \frac{\sqrt{3}}{4} (6 + y - y^2)^2 dy = \frac{625\sqrt{3}}{24} \quad \boxed{\text{C}}$$

$$20) \text{ Using the shell method with } p(x) = x^{\frac{a}{b}} \text{ and } r(x) = (a+b-x), \text{ Volume} = 2\pi \int_0^{a+b} x^{\frac{a}{b}} (a+b-x) dx \quad \boxed{\text{A}}$$

$$21) \text{ Area of region} = \int_{-1}^2 (x+1 - (x^2-1)) dx = \frac{9}{2} \text{ Centroid of region is } \bar{x} = \frac{2}{9} \int_{-1}^2 x(x+1 - (x^2-1)) dx = \frac{1}{2}$$

$$\bar{y} = \frac{2}{9} \int_{-1}^2 \left[ \frac{(x+1) + (x^2-1)}{2} (x+1 - (x^2-1)) \right] dx = \frac{3}{5} \text{ Point to line} = \frac{|1(1/2) + -1(3/5) + 8|}{\sqrt{1^2 + (-1)^2}} = \frac{79\sqrt{2}}{20}$$

$$\text{Volume} = 2\pi \frac{79\sqrt{2}}{20} \frac{9}{2} = \frac{711\pi\sqrt{2}}{20} \quad \boxed{\text{B}}$$

$$22) \text{ Volume} = 2\pi \int_1^e x \ln x dx = \frac{\pi(e^2 + 1)}{2} \quad \boxed{\text{B}}$$

$$23) \int_0^1 \tan x dx \approx .25 \tan(.25) + .25 \tan(.50) + .25 \tan(.75) + .25 \tan(1) \approx .823 \quad \boxed{\text{D}}$$

$$24) \text{ Area} = \frac{1}{2} \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2 \cos \theta)^2 d\theta = \frac{3\sqrt{3} + 4\pi}{2} \quad \boxed{\text{A}}$$

$$25) \text{ Area} = \int_{\frac{1}{4}}^{\frac{4}{3}} (x+3) - (4-3x) dx + \int_{\frac{7}{2}}^{\frac{7}{4}} (x+3) - (3x-4) dx = \frac{169}{72} + \frac{169}{36} = \frac{169}{24} \quad \boxed{\text{C}}$$

$$26) \text{ Area} = 2\pi \int_1^4 \frac{1+x}{\sqrt{x}} dx = \frac{40\pi}{3} \quad \boxed{\text{E}}$$

$$27) \text{ Volume} = \pi \int_0^1 (\sqrt{\arcsin x})^2 dx = \frac{\pi(\pi-2)}{2} \quad \boxed{\text{D}}$$

$$28) \text{ Volume} = \int_0^1 \frac{\left(\frac{4x^2}{\sqrt{2}}\right)^2}{2} dx = \int_0^1 4x^4 dx \frac{4}{5} = \quad \boxed{\text{C}}$$

$$29) \text{ Volume} = \pi \int_0^2 \left( \frac{1}{\sqrt{x^2 - 7x + 12}} \right)^2 dx \Rightarrow \text{Using partial fractions} = \pi \int_0^2 \left( \frac{1}{x-4} - \frac{1}{x-3} \right) dx = \pi \ln \left( \frac{3}{2} \right) \quad \boxed{\text{B}}$$

$$30) \text{ Area} = \int_0^{\frac{\pi}{4}} (\cos x - \sin x) dx = \sqrt{2} - 1 \quad \boxed{\text{C}}$$