

**TRIGONOMETRY - ALPHA**  
**National Mu Alpha Theta Convention 2003**

For all questions, choice E, "NOTA," means none of the above answers is correct.

1. Through what angle in radian measure must the following conic be rotated in order to eliminate the  $xy$  term? Round to the nearest thousandth.

$$24x^2 + 5xy + 4y^2 + 6x + 11y + 49 = 0$$

A: 0.663                      B: 0.681                      C: 0.122                      D: 0.245                      E: NOTA

2. For the function:  $y = 10\sin(6x) + 24\cos(-6x)$   
Let A equal the period of the function.  
Let B equal the amplitude of the function.  
What is  $A\pi^{-1} + B$ ?

A:  $\frac{157}{6}$                       B:  $\frac{79}{3}$                       C:  $\frac{77}{3}$                       D:  $\frac{155}{6}$                       E: NOTA

3. If the degree measure of the smallest positive angle between vectors  $\mathbf{a}$  and  $\mathbf{v}$  is in the form:  $\arccos \frac{A\sqrt{B}}{C}$  when the argument is in simple radical form, find  $A + B + C$

[ vector  $\mathbf{a} = (28,4)$  and vector  $\mathbf{v} = (12,18)$  ]

A: 167                      B: 139                      C: 173                      D: 145                      E: NOTA

4. Evaluate the determinant:
- $$\begin{vmatrix} \sin 30^\circ & \tan 45^\circ & \tan 30^\circ \\ \cos \frac{\pi}{6} & \cos \frac{\pi}{3} & \sin \frac{\pi}{2} \\ \cot 30^\circ & -\sec \pi & \sec 60^\circ \end{vmatrix}$$

A:  $\sqrt{3}$                       B:  $\frac{\sqrt{3}}{2}$                       C: 1                      D:  $\frac{1}{2}$                       E: NOTA

5. Which of the following angles is coterminal with  $\frac{123\pi}{456}$ ?

A:  $\frac{234\pi}{152}$                       B:  $-\frac{567\pi}{152}$                       C:  $-\frac{123\pi}{456}$                       D:  $-\frac{345\pi}{456}$                       E: NOTA

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6. Convert  $(13, -13)$  from rectangular into polar coordinates.
- A:  $(13\sqrt{2}, -\frac{5\pi}{4})$       B:  $(-13\sqrt{2}, -45^\circ)$       C:  $(-13\sqrt{2}, 495^\circ)$   
D:  $(13\sqrt{2}, \frac{3\pi}{4})$       E: NOTA
7. Find the area of the circle circumscribed about a triangle with side lengths 7, 5, and 8.
- A:  $\frac{49\pi}{3}$       B:  $49\pi$       C:  $\frac{7\pi}{2}$       D:  $\frac{7\pi}{3}$       E: NOTA
8. A wheel 4 feet in diameter is rotating at 80 rpm. Find the distance (in feet) traveled by a point on the rim in 1 second (to the nearest  $10^{\text{th}}$  of a second).
- A: 4.2 fps      B: 8.4 fps      C: 16.8 fps      D: 33.6 fps      E: NOTA
9. Find the area of a regular octagon with side length 12. Round to the nearest natural number.
- A: 1391      B: 174      C: 695      D: 348      E: NOTA
10. Add the values corresponding to the true statements:
- ( 5 )       $r = 5 - 5\sin\theta$  is a cardioid  
( -3 )       $\text{Arcsin}x + \text{Arccos}x = \pi$   
( 314 )      The point (3,4) is on the unit circle  
( 36 )      The phase shift of  $y = 5\cos(6x+12)$  is 2
- A: 38      B: 33      C: 316      D: 319      E: NOTA
11. Find the hundreds place of  $(6 + 6i\sqrt{3})^6$
- A: 6      B: 7      C: 8      D: 9      E: NOTA
12. If  $\sinh x = \frac{e^x - e^{-x}}{2}$  and  $\cosh x = \frac{e^x + e^{-x}}{2}$ . Then find  $\cosh^2 4 - \sinh^2 4$ .
- A:  $\frac{1}{2}$       B: 1      C: 2      D: e      E: NOTA

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13. Find the measure of the largest acute angle in the triangle formed by the diagonal of a cube, the diagonal of one of its faces, and the included edge. Round to the nearest tenth.
- A:  $35.3^\circ$       B:  $54.7^\circ$       C:  $42.5^\circ$       D:  $47.5^\circ$       E: NOTA
14. If  $\sin 165^\circ = a(\sqrt{b} - \sqrt{c})$  where a is positive and the right side of the equation is in simple radical form, find  $bca^{-1}$
- A: 3      B: 12      C: 96      D: 48      E: NOTA
15. Find ( Range of  $y = \text{Arccsc}x$  )  $\cap$  ( Angle measures that make  $\csc\theta$  negative )
- A:  $\left(-\frac{\pi}{2}, 0\right)$       B:  $\left[-\frac{\pi}{2}, 0\right]$       C:  $\left[-\frac{\pi}{2}, 0\right)$       D:  $\left[-\frac{\pi}{2}, 0\right]$       E: NOTA
16. How many petals are in the graph of  $r = 3\cos 4\theta$ ?
- A: 3      B: 4      C: 6      D: 8      E: NOTA
17. Find the sine of the angle (between 0 and 180 degrees) between vectors  $\overrightarrow{AB}$  and  $\overrightarrow{AC}$  if  $A = (2,3)$   $B = (4,-2)$  and  $C = (-3, 6)$   
Round to the nearest thousandth.
- A: 0.204      B: 0.605      C: 0.395      D:  $-0.796$       E: NOTA
18. Find the product of all solutions in the interval  $(0, 2\pi]$
- $$\tan^2 x - 2\tan x \sin x = 0$$
- A: 0      B:  $\frac{2}{3} \pi^4$       C:  $\frac{10}{9} \pi^4$       D:  $\frac{70}{9} \pi^4$       E: NOTA
19. Which of the following statements are true?
- I       $\sin(90^\circ - \theta) = \csc\theta$   
II      The imaginary axis in the Argand plane is horizontal.  
III       $r^2 = 26\cos 2\theta$  is a horizontally oriented lemniscate.
- A: III only      B: II, III only      C: I, III only      D: I, II only      E: NOTA

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20. Evaluate:  $\lim_{x \rightarrow 0} \frac{\sin x}{x}$
- A: 0      B: 1      C: e      D: DNE      E: NOTA

21. Simplify:  $\frac{\cos\theta}{1 + \sin\theta} + \frac{1 + \sin\theta}{\cos\theta}$

(The domain for all answer choices exists such that  $\sin\theta \neq -1$  and  $\cos\theta \neq 0$ )

- A:  $2\sec\theta$       B:  $2\cos\theta$       C:  $\sin^2\theta$       D:  $\sin 2\theta$       E: NOTA
22. A charter bus is traveling west on alligator alley at 72 miles per hour (mph). While looking out the window at an angle of  $60^\circ 31' 48''$  north of the bus's course, Christine sees her friend Synapse on an airboat that is traveling east, 3 miles from the highway. One minute later, the bus and airboat pass each other. How fast is the airboat traveling? Round to the nearest thousandth.
- A: 13.314 mph      B: 12.389 mph      C: 29.715 mph  
D: 30.609 mph      E: NOTA

23. If  $\sin A = \frac{4}{5}$  and  $\cos B = -\frac{12}{13}$ , find  $\tan(A-B)$ . Angle A is in the second quadrant, angle B is in the third.

- A:  $-\frac{63}{16}$       B:  $\frac{9}{8}$       C:  $\frac{33}{16}$       D:  $\frac{179}{168}$       E: NOTA

24. Find the eccentricity of the following conic:

$$r = \frac{5}{2 - 3\cos\theta}$$

- A:  $\frac{2}{5}$       B:  $\frac{3}{5}$       C:  $\frac{2}{3}$       D:  $-\frac{2}{3}$       E: NOTA
25. If  $\sin\theta$  is in the simple radical form  $\frac{\sqrt{A} - \sqrt{B}}{C}$ , find  $A + B + C$  where  $\theta$  = the angle between the hour and minute hands of a clock at 6:36 p.m.
- A: 10      B: 15      C: 17      D: 23      E: NOTA

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26. What is the mode of the following set of numbers?

$$\left\{ \cot \frac{5\pi}{4}, {}_6C_5, 3\sec \frac{5\pi}{3}, \cos 90^\circ, 6! \right\}$$

- A: 0                  B: 1                  C: 6                  D: 720                  E: NOTA
27. Find the tangent of the acute angle between  $y = 6x - 3$  and  $60x + 12y = 9$
- A:  $\frac{1}{31}$                   B:  $\frac{11}{29}$                   C:  $-\frac{1}{31}$                   D:  $-\frac{11}{29}$                   E: NOTA
28. For what value of J are  $\langle 3, 2, -5 \rangle$  and  $\langle J, 6, 9 \rangle$  perpendicular vectors?
- A: 19                  B: 16                  C: 15                  D: 11                  E: NOTA
29. How many of the following are even functions?

$$y = \sin x$$

$$y = |x|$$

$$y = 32x^4$$

$$y = \sec x$$

$$y = x^2 + 1$$

$$x^2 + y^2 = 36$$

- A: 3                  B: 4                  C: 5                  D: 6                  E: NOTA
30. What is the area of a parallelogram with adjacent sides of lengths a and b with acute angle  $\theta$  as the included angle?
- A: ab                  B:  $a\sin\theta$                   C:  $ab\cos\theta$                   D:  $ab\tan\theta$                   E: NOTA