Mu Alpha Theta National Convention 2003
Alpha Analytic Geometry Solutions

1. **A)** - Although Euclid (c. 300 BC) preceded Appolonius (c 262-200 BC), Appolonius is considered the father of analytic geometry for his extensive works in conics among other topics.

2. **B)** - The eccentricity of all parabolas is 1.

3. **A)** – For a plane with equation $Ax + By + Cz + D = 0$ and point (x_1, y_1, z_1), the distance from plane to point is given by
$$d = \frac{Ax_1 + By_1 + Cz_1 + D}{\sqrt{A^2 + B^2 + C^2}}$$

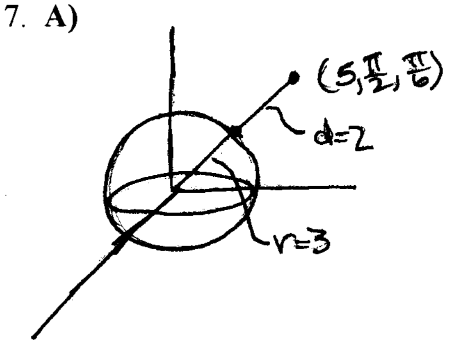
4. **C)** – For an ellipse with foci lying on the y-axis ($\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$), the eccentricity is defined as
$$e = \frac{c}{a} = \frac{\sqrt{a^2 - b^2}}{a}$$

5. **E)** – For hyperbola with equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, the asymptotes have slopes of $\pm\frac{b}{a}$.

In this case, $\pm\frac{b}{a} = \pm\frac{3}{2}$.

6. **A)** – For a parabola with vertex at the origin and focus lying on the y-axis at $(p,0)$, the equation for the parabola can be written as $y^2 = 4px$.

7. **A)**



8. **B)**
$$A_{ellipse} = \pi ab$$
$$A_{circle} = \pi a^2$$
$$\frac{A_{ellipse}}{A_{circle}} = \frac{b}{a}$$

9. **D)** – the counterclockwise angle, $\theta = 90 - \arctan m$ is found where m is the positive root of $hm^2 + (a-b)m - h = 0$ from the equation:
$$ax^2 + 2hxy + by^2 + \ldots = 0$$

10. **C)**
$$4\sin\theta\cos\theta = 2\sin(2\theta)$$
For $r = a\sin(b\theta)$ where b is even, the number of petals, $p = 2b$.

11. **B)** - Each of the fifth roots of -2 lie on a circle around the origin with radius $r = \sqrt[5]{2}$, where the absolute value of each point on the circle is $\sqrt[5]{2}$. Therefore, the sum of 5 fifth roots is $5\sqrt[5]{2}$.

12. **A)** - The length of the $lr = \frac{2b^2}{a}$.
The distance between the latera recta
$d = 2c = 2\sqrt{a^2 - b^2}$.
$$A = lr \cdot 2c = \frac{8}{3} \cdot 2\sqrt{5}$$

13. **D)** – centers:
circle 1: (1,1)
circle 2: (-3,-2)
$$d = \sqrt{(1+3)^2 + (1+2)^2} = 5$$

14. **B)** sphere 1: $r = \sqrt{3}$
cube 1: $s = 2$
sphere 2: $r = 1$ etc ....
$$ratio_{radii} = \frac{1}{\sqrt{3}}$$
$$\sum radii = \frac{\sqrt{3}}{1 - 1/\sqrt{3}} = \frac{3}{\sqrt{3} - 1}$$

15. **B)**
$$x = \frac{-b}{2a} = -3$$
$$y = (-3)^2 + 6(-3) + 11 = 2$$

16. **A)** – The line must have a slope, $m = 1$. It is therefore tangent to the circle at the points (-2,2) and (2,-2). The possible answers are $y = x \pm 4$.

17. **C)**
$\det = (\sin x + 1)(1 - \sin x \cos x)$
when $x = \frac{3\pi}{2}$, $\sin x = -1$
and $\det = 0$

18. **E)**

$$r_{circum} = \frac{abc}{4 \cdot Area} = \frac{5}{2}$$

$$r_{in} = \frac{Area}{semi-perimeter} = 1$$

19. **D)** "Green's Theorem" for polynomials...

	0	0	
0	2	2	0
10	5	3	6
12	4	-3	-15
-3	1	-4	-16
0	0	0	0
19			-25

$$A = \frac{|19-(-25)|}{2} = 22$$

20. **D)** – For a cycloid with equations
$x = a(t - \sin t)$ and
$y = a(1 - \cos t)$, the period
$T = 2a\pi = 4\pi$

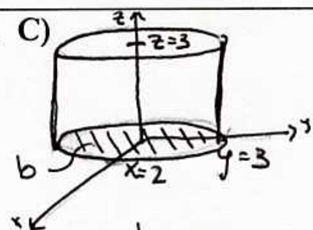
21. **A)** - Take two vectors formed at the intersection of the two lines (a good pair are $a = \langle 2,3 \rangle$ and $b = \langle 1,5 \rangle$)

Now, $\cos\theta = \frac{a \cdot b}{\|a\|\|b\|} = 45°$

22. **D)** – the perpendicular vector can be found by computing the cross product $\langle 1,2,3 \rangle \times \langle 4,5,6 \rangle = \langle -3,6,-3 \rangle$ so $\langle -1,2,-1 \rangle$ will also be perpendicular.

23. **B)** - The ratio of areas is equal to the determinant of the transform matrix, in this case 5. To see this, multiply each point like so:
$\begin{bmatrix} 1 & -2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$, and find the area of the triangle given by the resulting points.

24. **C)**



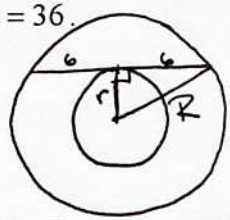
$A = bh,$
$b = 2 \cdot 3 \cdot \pi,\ h = 3$
$A = 18\pi$

25. **D)** – The equation can be rewritten as $(2x - y + 3)^2 = 0$, which drawn as one line.

26. **C)** From the definition of a parabola.

27. **C)** – A parabola can be written in polar for as $r = \frac{2p}{1 \pm \cos\theta}$ or $\frac{2p}{1 \pm \sin\theta}$, $p \neq 0$.

28. **D)** – The area of the annulus between the two circles is given by $A = \pi(R^2 - r^2)$. From the triangle, $(R^2 - r^2) = 36$.



29. **B)** – The five platonic solids are a tetrahedron, cube, octahedron, dodecahedron, and icosahedron, having faces of 4, 6, 8, 12, and 20, respectively.

30. **C)** – For the most general second degree equation with six variables, it would require six points to assure that you could solve for each variable.