Question 1
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If $A = \prod_{i=1}^{16} \frac{i}{i+1}$

$B = \sum_{i=-8}^{8} (2i-1)$

C = the constant term in the expansion of $(x^2 - \frac{2}{x^6})^8$

Find ABC.

Question 2
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Given $\frac{6}{\sqrt[3]{4} - \sqrt[3]{2}} = 3(\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c})$ and $(2i - 1)^5 = d + ei$, find a + b + c - d - e.

Question 3
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Given $g(x+\pi) = 3\sin(2x+\pi) - 4$ and $h(x) = 2\tan(x-\pi) + 3$ find $\frac{A-D}{B+C}$ if

A = amplitude of g(x)
B = period of h(x)
C = phase shift of h(x). $(0 \le C < 2\pi)$
D = maximum value of g(x)

Question 4
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In a coordinate plane, find the area of the region formed by the intersection of $x > 0, y < 1,$ and $x^2 + y^2 < 4y$.

Question 5
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Find the value of (A +C)B given

$A = i^{0!} + i^{1!} + i^{2!} + i^{3!} + \ldots + i^{100!}$ $(i = \sqrt{-1})$

B = absolute value of the reciprocal of 4 + 3i

C = solution to $\frac{1}{\sqrt{4}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{6}} + \frac{1}{\sqrt{6}+\sqrt{7}} + \ldots + \frac{1}{\sqrt{C}+\sqrt{C+1}} = 10$

Question 6
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Find (A / B) / C given,

$A = \frac{2a+3b}{4b+3c}$ if a: b: c = 3: 1: 5

B = exponent such that $\sqrt{\frac{a}{b}\sqrt{\frac{b}{a}\sqrt{\frac{a}{b}}}} = (\frac{a}{b})^B$, for $ab \neq 0$.

.C = $m^3 + n^3$ given m + n = 3 and $m^2 + n^2 = 6$

Question 7
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Given

a = solution to $\left(\log_a(2a)\right)(\log a) = 3$

b = the least integral value such that $\log_2 3, \log_2 7,$ and $\log_2 b$ can be the sides of a triangle

c = the positive real value of c for $\log_x(\log_3 c^2) = 2$ if $x = \log_3 c$

Find a + b – c

Question 8
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Given the three digit numbers AB4; B03; B3C; and BA1 form an arithmetic sequence find A + B + C.

Question 9
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In degrees, what is the sum of the degree measures of all the angles x, -720 < x < 360, for which

$$(2^{\sin^2 x})(2^{\tan^2 x})(2^{\cos^2 x}) = 2^2$$

Question 10
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How many integral values in the intersection of the Real domains of the following functions?

$f(x) = \sqrt{x^2 - 4}$, $g(x) = \frac{1}{\sqrt{9 - x^2}}$, and $h(x) = \sqrt{\frac{2x}{5 - x}}$

Question 11
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Let A, B, and C be the solutions to each of the following problems, then find $\frac{A+B}{C}$.

A: A plane flew from city X to city Y at a rate of 380 mph and returned from Y to X at a rate of 420 mph. What was the average rate of speed in mph for the round trip?

B: Tom, Dick, and Harriet were born on January 1 in consecutive years. In five years, five times Harriet's age will be 26 more than twice the sum of Dick's and Tom's age at that time. Harriet is the oldest and Tom the youngest. How old will Tom be next year?

C: A 25-foot ladder rests against a building such that the foot of the ladder is 7 feet from the building. If the top of the ladder slipped down an additional 4 feet, how many feet does the foot of the ladder slide?

Question 12
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Find sin A, given A is the largest angle in a triangle with sides 4, 5, & 7.

Question 13
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What is the least possible distance between the graphs of the equations $x^2 + y^2 = 1$ and $x^2 + y^2 -10x - 24y +168 = 0$.

Question 14
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Find the product of the two unique square roots of 9i.

Question 15
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Find the distance between the polar coordinates $(6\sqrt{2}, \frac{\pi}{4})$ and $(4, \frac{3\pi}{2})$.

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1. $A=\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\cdots\frac{16}{17}=\frac{1}{17}$

$B = -17-15-13-\ldots+13+15=\frac{(17)(-2)}{2}=-17$

$C = {}_8C_2\,(x^2)^6(\frac{-2}{x^6})^2 = 112$

ABC = -112

2.

$$\frac{6}{\sqrt[3]{4}-\sqrt[3]{2}}\cdot\frac{\sqrt[3]{16}+\sqrt[3]{8}+\sqrt[3]{4}}{\sqrt[3]{16}+\sqrt[3]{8}+\sqrt[3]{4}}=$$

$$\frac{6(\sqrt[3]{16}+\sqrt[3]{8}+\sqrt[3]{4})}{4-2}=3(\sqrt[3]{16}+\sqrt[3]{8}+\sqrt[3]{4})$$

a=16, b=8, c=4

$(2i-1)^5 = -41-38i$

d=-41, e=-38

a+b+c-d-e= 107

3. Given $g(x+\pi)=3\sin(2x+\pi)-4$ then

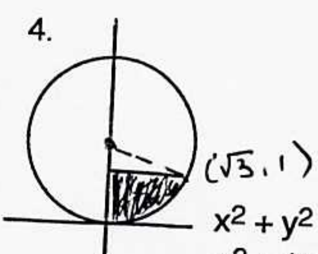
$g(x)=3\sin(2(x-\pi)+\pi)-4$

$=3\sin 2(x-\frac{\pi}{2})-4$

$A=3, B=\pi, C=\pi, D=-1$

$\frac{A-D}{B+C}=\frac{4}{2\pi}=\frac{2}{\pi}$

4.



$x^2 + y^2 - 4y < 0$

$x^2 + (y-2)^2 < 4$

Point of intersection of circle and y=1 is $(\sqrt{3},1)$.

Area of sector = $\frac{1}{6}(4\pi)=\frac{2\pi}{3}$,

Area of triangle = $\frac{\sqrt{3}}{2}$.

Contained area=$\frac{2\pi}{3}-\frac{\sqrt{3}}{2}$

5. $i^{0!}+i^{1!}+i^{2!}+i^{3!}+i^{4!}+i^{5!}+\ldots+i^{100!}=$

$i \;+ i \;-1 \;-1 \;+1 \;+1 +\ldots+ 1 = 95+2i$

$\left|\frac{1}{4+3i}\right|=\left|\frac{4-3i}{25}\right|=\frac{1}{5}$

For C, rationalize the denominators of each rational expression:

$$\frac{\sqrt{4}-\sqrt{5}}{-1}+\frac{\sqrt{5}-\sqrt{6}}{-1}+\ldots+\frac{\sqrt{c}-\sqrt{c+1}}{-1}=10$$

$\sqrt{4}-\sqrt{c+1}=-10$

$\sqrt{c+1}=12$

C=143

Therefore, $(A+C)B=\frac{238}{5}+\frac{2}{5}i$

6. Using ratio, a=3b and c=5b.

$A=\frac{2a+3b}{4b+3c}=\frac{2(3b)+3b}{4b+3(5b)}=\frac{9}{19}$.

Change each radical into exponential form

$$\sqrt{\frac{a}{b}\sqrt{\frac{b}{a}\sqrt{\frac{a}{b}}}}=\sqrt{\frac{a}{b}\sqrt{(\frac{a}{b})^{-1/2}}}=\sqrt{(\frac{a}{b})^{\frac{3}{4}}}=(\frac{a}{b})^{\frac{3}{8}}$$

Given m+n=3 then $m^2+2mn+n^2 = 9$ and since $m^2+n^2=6$ means mn=3/2.

Factor $m^3+n^3=(m+n)(m^2-mn+n^2)$ and substitute$\Rightarrow$ $m^3+n^3=27/2$.

$(A/B)/C=\frac{16}{171}$

7. $(\log_a 2a)(\log a) =$

$(\frac{\log 2a}{\log a})(\frac{\log 2}{\log 10})=\log 2a=3 \Rightarrow a=500$.

For least value $\log_2 7-\log_2 3<\log_2 b\Rightarrow b>\frac{7}{3}$.

Solve for x then substitute. $\log_x(2x)=2\Rightarrow$ $x^2=2x\Rightarrow x=2$ and 0(bad)$\Rightarrow 2=\log_3 c\Rightarrow$ c=9. Therefore, a+b-c=500+3-9=494.

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8. From looking at list B=A+1, the common difference must have a units digit of 9 (which means C=2) Using substitution my new numbers are 110A+14, 100A+103, 100A+132, and 110+101. The difference between consecutive terms equals the common difference$\Rightarrow$A=6 and B=7. A+B+C=15.

9. When multiplying like base numbers add exponents$\Rightarrow 1+\tan^2 x=2 \Rightarrow \tan x = \pm 1$ $\Rightarrow x = 315+225+135+45-45-135-225-315-405-495-585-675=-2160$. -2160

10. Domain for f(x), $x^2-4 \geq 0 \Rightarrow x \geq 2$ or $x \leq -2$, domain for g(x), $9-x^2>0 \Rightarrow -3<x<3$, domain for h(x), $\frac{2x}{5-x} \geq 0 \Rightarrow 5 > x \geq 0$. Integers in common is limited to the value 2. Thus, only 1 integer.

11. $A=\frac{2(380)(420)}{(380+420)}=399$

Let Tom=x, Dick=x+1, and Harriet=x+2$\Rightarrow$ $5(x+2+5)-26=2((x+5)+(x+1+5)) \Rightarrow x=$ 13

Next year Tom will be 14

Original triangle formed 7-24-25. Slide down 4 feet and triangle becomes 15-20-25. Ladders moves an additional 8 ft.

$\frac{A+B}{C}=\frac{413}{8}$.

12. Find area using Heron's $A=\sqrt{8(4)(3)(1)}=4\sqrt{6}$.

Area also can be found by $\frac{1}{2}$absinC=

$\frac{1}{2}(4)(5)\sin C=4\sqrt{6} \Rightarrow \sin C=\frac{4\sqrt{6}}{10}=\frac{2\sqrt{6}}{5}$.

13. Find distance between centers (0,0) and (5,12) less the radii ($r_1=1$ and $r_2=1$). $\Rightarrow 13-(1+1)=11$

14. Let $x=\sqrt{9i} \Rightarrow x^2=9i \Rightarrow x^2-9i=0 \Rightarrow$ product of roots=-9i.

15. Change to rectangular form: (6,6) and (0,-4). Distance = $\sqrt{136}=2\sqrt{34}$.