Theta Number Theory Answer Key

c 1) $\frac{2+3+5+7+11+13+17+19+23+29}{10} = \frac{129}{10} = 12.9$

b 2) $1^1 + 3^2 + 5^3 = 1+9+125 = 135$

b 3) 36 has factors 1, 2, 3, 4, 6, 9, 12, 18 + 36

c 4) 1, 4, 9, 16, 25, 36 ... $(31)^2$ $\frac{16}{31} \sim 52\%$

b 5) $3(1^{st}) - (2^{nd}) = 3(2) - 8 = -2$

d 6) Find LCM for 6, 8 + 9
$2 \cdot 3, 2^3, 3^2$ $2^3 \cdot 3^2 = 72$

a 7) Find GCF of 24 and 90
$2 \cdot 2 \cdot 2 \cdot 3$ $2 \cdot 3 \cdot 3 \cdot 5$ $\Rightarrow 2 \cdot 3 = 6$

d 8) First it must be odd 61, 63, 65 or 67
63 is divisible by 3; 65 divisible by 5, 61 is not a choice so 67

c 9) Factors of 279 are 1, 3, 31, 279 so of the ones listed choose 31.

b 10) $\sqrt{8}$ and $\sqrt{80}$ $\sqrt{8} < \sqrt{9}$ + $\sqrt{80} < \sqrt{81}$
$\sqrt{9} = 3$, $\sqrt{81} = 9$ so 3, 4, 5, 6, 7, 8 are in between

b 11) $\sqrt{65} - \sqrt{63} = 0.1250038...$ so B 0.13

b 12) Friday the 13ths can only occur when the sum of the days from the month with the Fri. 13 to month before the next month occurence is a multiple of 7. For example if Fri. 13th occurs in April then it will occur again in July because 30+31+30 = 91

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d 13). Digits repeat 2, 4, 8, 6 ... $\frac{2391}{4} = 597$ remainder 3 so $\underline{8}$.

a 14) odd + odd = even so 2 must be a factor + 2 is smallest prime

c 15) Test some numbers with lots of factors

Consider $100 = 2^2 \cdot 5^2 \Rightarrow (1+2)(1+2) \Rightarrow 9$ factors

$99 = 3^2 \cdot 11 \Rightarrow (1+2)(1+1) \Rightarrow 6$ "

$96 = 2^5 \cdot 3^1 \Rightarrow (5+1)(1+1) \Rightarrow 12$ "

$72 = 2^3 \cdot 3^2 \Rightarrow (1+3)(1+2) \Rightarrow 12$ "

d 16) $1^6 = 1 \to 1$

$2^6 = 64 \to 10 \to 1$

$3^6 = 729 \to 18 \to 9$

$4^6 = 4096 \to 19 \to 10 \to 1$

$5^6 = 15625 \to 19 \to 10 \to 1$

pattern 1, 1, 9, 1, 1, $\underline{9}$

d 17) At least one zero for each multiple of 5; $\Rightarrow 10$

2 multiples 25 + 50 add 2 zeros each $+ \underline{2}$

12

b 18) Square is best "rectangle" choice so $4x = 10$ $x = 2.5$

so $d^2 = (2.5)^2 + (2.5)^2 = 12.5$

$d = \sqrt{12.5} \approx 3.536$

d 19) $2^3 \cdot 5^3 = 1000$ so form $2^m, 5^n$ $m + n \in \{0, 1, 2, 3\}$

$4 \cdot 4 = 16$ factors

a 20) girls = 18 + boys boys + girls = 44 => boys = 13

team must have at least 1 boy + 1 girl => max number of teams can be 13

a 21) The idea would be to test two numbers which are closest together. Of the choices $6234 - 5987 = 257$ is smallest

e 22) If we pair $\underbrace{1-2}_{-1} + \underbrace{3-4}_{-1} + \ldots + \underbrace{97-98}_{-1} + 99 \Rightarrow$

$-49 + 99 = 50$

d 23) If odd = x, then $x^2 - 29$ = even and > 2

Test => x = even

$6^2 - 29 = \underline{7}$; $8^2 - 29 = 35$; $10^2 - 29 = \underline{71}$; $12^2 - 29 = 115$; $14^2 - 29 = \underline{167}$

3 out of 9 choices => 1/3

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b 24) $5 \cdot 13 = 65$ Start multiplying 65 by values and 16 is smallest to be >1000; but is ÷ by 4. First factor to meet criteria afterwards is 17 so $65(17) = 1105$

a 25) $(2-1) + (3-1) + \dots + (10-1)$
$1 + 2 + \dots + 9 = 45$

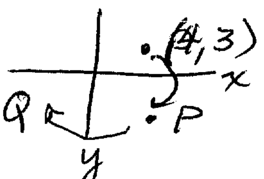
b 26) Eliminate c, d, e all ≥ 1 consider $\frac{6}{7}$ or $\frac{6}{8}$

d 27) the product will produce 5 decimal places and 5 zeros so $(1998)^2$

d 28) $2^{1999} \cdot 5^{1999} \cdot 5^2 = \underbrace{10^{1999}}_{\text{zeros}}(25)$ so $2+5=7$

a 29) 1 less than a multiple of 5 ⇒ ending # is 4 or 9 with multiples of 4, reject 4 + consider form $10d+9$
Only 9, 29, 49, 69 + 89 are 1 more than multiple of 4, and only 29 + 89 are prime. Their sum = 118

a 30)



$Q(-4,-3)$ sum $= -7$

b 31) $(\text{odd})^2 = 1 +$ multiple of 4 $\quad (2n+1)^2 = 4n^2 + 4n + 1$
⇒ 2 more than a multiple of 4. Only one = 1998

d 32) $d_1 + d_2 + \dots + d_n = 1170$
$\frac{1}{d_1} + \frac{1}{d_2} + \dots + \frac{1}{d_n}$ the LCD would have to be 360
so $1170/360 = \frac{117}{36}$

b 33) $7 + e + f = 18$
⇒ $e + f = 11$
$e + f + g = 18$
$11 + g = 18$
⇒ $g = 7$ (left of x)
$j + k + 8 = 18$
⇒ $j + k = 10$
$i + j + k = 18$
⇒ $i = 8$ (right of x)
$7 + x + 8 = 18$
$\underline{x = 3}$

c 34) For 30 (multiples of) $= 2^x 3^y 5^z N$; x, y + z are positive
# of divisors $= (x+1)(y+1)(z+1)n$; we want 36
so $y \geq 1$, $z \geq 1$ then $x \in \{1, 2, 3, 5, 8\}$ Test $2^2 \cdot 3^2 \cdot 5 \cdot 7 = \underline{1260}$ smallest

b 35) Theorem (Kummer) states odd numbers in row of Pascal's Δ $= 2^r$ where $r = $ # of 1's in binary expansion
if $n = 100$ then $100_{10} = 1100100_2$ ⇒ 1 occurs 3 times
so $2^3 = 8$