

**2002 National Mu Alpha Theta Convention
Theta Geometry Test**

NOTA means "None Of The Above"

In all questions, units are not required. The test writer has not tried to trick you by changing feet into yards, or anything else likewise. For example, if asked for an area of a circle with radius 4 feet, assume all answers are in feet squared.

1. Given: $\triangle ABC$ with $\angle C = 55^\circ$, and $\angle B = 65^\circ$. Extend \overline{AB} 3 units to a point D, so that A is between D and B; extend \overline{AC} 3 units to a point E, so that A is between E and C. Find the area of triangle ADE in square units.

- A. $\frac{9\sqrt{3}}{4}$ B. $9\sqrt{3}$ C. Not Enough Information D. π E. NOTA

2. If two planes intersect, then their *intersection* can be

I. a line. II. three noncollinear points. III. two intersecting lines.

- A. I only B. II only C. III only D. I and II only E. NOTA

3. Consider a regular nonagon. Find the number of diagonals plus the number of degrees in the sum of the interior angles plus the number of degrees in the sum of the exterior angles (one at each vertex) plus the number of sides plus one.

- A. 1653 B. 1654 C. 1655 D. 1656 E. NOTA

4. Micah is running around a track on the coordinate plane. The track has equation $x^2 + y^2 - 6x - 14y - 111 = 0$. If Micah runs two laps around the track, how far has he traveled?

- A. 13π B. 52π C. 169π D. 26π E. NOTA

5. A rectangular picture of width 30 centimeters and height 20 centimeters is framed in a rectangular frame 3 centimeters wide. What is the area of the frame in square centimeters?

- (A) 288 cm^2 (B) 300 cm^2 (C) 336 cm^2 (D) 352 cm^2 (E) NOTA

6. In a rhombus, the product of the lengths of the diagonals measured in centimeters is 65, and the sum of the lengths of the diagonals measured in centimeters is 20. What is the area of the rhombus in square centimeters?

- (A) 10 cm^2 (B) $\frac{45}{2} \text{ cm}^2$ (C) 30 cm^2 (D) $\frac{65}{2} \text{ cm}^2$ (E) NOTA

7. For what positive radius "r" will the number of units in the circumference of a circle be equal to the number of square units in its area?

A. 1 B. 2 C. 3 D. π E. NOTA

8. A gold sphere of radius $\sqrt{e\pi}$, where "e" is the natural base, is melted into a right circular cone of radius $\sqrt{5}$, and no gold is lost in the process. What is the slant height of the cone, rounded to the nearest hundredth?

A. 6.65 B. 7.02 C. 19.96 D. 20.09 E. NOTA

9. A circle in the coordinate plane is drawn using the equation $x^2 + y^2 - 6x = 0$. Find the length of one of the tangents drawn from the point $(-2, 5)$ to the point of tangency.

A. $\sqrt{41}$ B. $3\sqrt{2}$ C. $5\sqrt{2}$ D. 6.5 E. NOTA

10. Point A is outside a circle. Two line segments are drawn from A to the circle: one is tangent to the circle at B, and one is a secant that cuts the circle at C and ends at D (also a point on the circle). If $AC = 2$, $AB = 3$, and $BD = 5$, find the area of $\triangle ABD$ to the nearest thousandth.

A. No such triangle exists B. 6.666 C. 5.166 D. 7.211 E. NOTA

11. An elliptical dartboard has a major axis of length 16 feet and minor axis of length 14 feet. A square of side length 4 feet shares the same center as the ellipse, and inscribed in it is a circle. Aneesh throws a dart at the dartboard and is equally likely to hit any point on the dartboard. Assuming he does hit the board, what is the probability that he hits in the area inside the square, but outside the circle (round to the nearest hundredth).

A. 0.00 B. 0.07 C. 0.02 D. 0.01 E. NOTA

12. In triangle HIJ, angle J measures 105 degrees, angle H measures 30 degrees, side $JI = 4$. What is the length of side HJ?

A. $6\sqrt{3}$ B. $2\sqrt{2}$ C. $4\sqrt{2}$ D. 3.46 E. NOTA

13. In Euclidean geometry, which of the following statements is (are) true?

- I. Given two distinct points, there is exactly one line which contains both points.
- II. Given three distinct points, there is exactly one plane that contains all three points.
- III. Given a line A, and a point B not on line A, there is exactly one line through point B parallel to line A.

(A) I only (B) I & II only (C) I & III only (D) I, II, & III (E) NOTA

14. A regular n-gon has interior angles of 171° . What is the value of n?

(A) 40 (B) 42 (C) 45 (D) 60 (E) NOTA

15. What is the surface area of a sphere whose volume is 288π ?

(A) 27π (B) 36π (C) 64π (D) 96π (E) NOTA

25. Let there be four points in the Cartesian plane D(-2, -5), E(2.5, -1.5), F(3, 8), and G(-8, -3). Let the line through E and G be represented by the equation $Ax + By = C$, where A, B, and C are relatively prime integers ($A > 0$). Let the midpoint of D and E be (H, J). Let the distance DG be represented by K. Let the slope of the line through F and E be L. Find $\frac{AC}{HJ} - \frac{BK}{L}$.

- A. $16 + \frac{14\sqrt{10}}{19}$ B. $-16 + \frac{14\sqrt{10}}{19}$ C. $16 - \frac{14\sqrt{10}}{19}$ D. $-16 - \frac{14\sqrt{10}}{19}$ E. NOTA

26. A square with 25-centimeter sides is inscribed in a circle. What percentage of the circle's area lies within the square?

- (A) $\frac{100(\pi - 2)}{\pi}$ (B) $\frac{100}{\pi}$ (C) $\frac{200}{\pi}$ (D) $\frac{100(\pi - 1)}{2\pi}$ (E) NOTA

27. In triangle ABC which has a right angle at C, $AC = 8$, $BC = 21$. What is $(\sin B)^2 + (\cos B)^2$?

- A. $\frac{29}{505}$ B. $\frac{29\sqrt{505}}{505}$ C. 2 D. 1 E. NOTA

28. Find the area of a triangle with vertices at coordinates (-5, 2), (8, 3), and (1, 5), and add it to the area of a pentagon with coordinates (4, 8), (1, -4), (-2, 5), (5, 2), and (-6, 0). Give your answer in square units.

- A. 62.5 B. 88.5 C. 57 D. 70.5 E. NOTA

29. Find the volume of a regular icosahedron with edge length $\sqrt{5}$.

- A. $\frac{125 + 75\sqrt{5}}{12}$ B. $\frac{175 + 75\sqrt{5}}{4}$ C. $\frac{25 + 15\sqrt{5}}{12}$ D. $\frac{125 + 75\sqrt{5}}{4}$ E. NOTA

30. Find the volume of the solid when the region bounded by the x-axis, the y-axis, and the line $3x + 2y = 12$ is revolved around the x-axis.

- A. 144π B. 48π C. 32π D. 96π E. NOTA

Work the following tiebreaker in the white portion on the back of the scantron sheet.

Given regular polygon ABCDEFGHIJKLMN with each side length 4 units. Find the area of triangle GHI, rounded to the nearest hundredth.