Theta Equations and Inequalities

National Mu Alpha Theta Convention 2002

1. A $4y(2 + b) - b(3y - 1) = 5b$;
   $8y + 4by - 3by + b = 5b$;
   $y = \frac{4b}{8+b}$
2. C $x^2 + 3x - 10 = 0$; $b^2 - 4ac = 9-4(-10)=49$
3. C $|2-3x| < 4$; $-4 < 2 - 3x < 4$; $\frac{-2}{3} < x < 2$
   $\left(\frac{-2}{3}, 2\right)$
4. E
5. B Solving the system $3x + y = 10$ and $x - 3y - 10 = 0$ you get $(4,-2)$ $x+y = 4-2=2$
6. A Percent of decrease =
   $\frac{90-75}{90} = \frac{15}{90} = .1\bar{6} = 16\frac{2}{3}\%$
7. B $.4(10in.) = \frac{x}{6}(24in.)$; $x=1$
8. B $6(4) = 5h$; $h = 4.8$
9. C $a^3 = 7$ $4a^6 = 4(a^3)^2 = 4(7)^2 = 196$
10. B $2x + 3 + \sqrt{29-4x} = 0$
    $\sqrt{29-4x} = -2x - 3$; $29-4x=4x^2+12x+9$
    $4x^2+16x-20=0$; $4(x^2+4x-5)=0$; $x=-5$, 1 but must reject 1; only solution is -5.
11. D
12. D $2x^4 - x^3 - 4x^2 + 10x - 4 = 0$ factors into $(x-1-i)(x-1+i)(2x-1)(x+2)=0$ therefore there are 2 rational and 2 imaginary roots.
13. B
14. D $f(x) = \frac{2x^3 + 15x^2 + 34x + 18}{x^2 + 5x + 4}$ has vertical asymptotes where $x^2 + 5x + 4 = 0$; $x = -1$, $x = -4$ The slant asymptote $y = 2x+5$ is found by dividing the numerator by the denominator.
15. E if $px^3 + px + q$ is divided by $x - 1$, the remainder is 3; if $px^3 + px + q$ is divided by $x + 1$, the remainder is -7. This implies $p(1)^3+p(1)+q = 3$ and $p(-1)^3 + p(-1) + q = -7$ which means $p + p + q = 3$ and $-p - p + q = -7$ Solving the system, $2q = -4$; $q = -2$ and $p = 2.5$
16. D Let r_1 and r_2 are the roots of the equation $ax^2 + bx + c = 0$, then
    $r_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$, $r_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$
    then $(r_1 - r_2)^2 =$
    $\left(\frac{-b + \sqrt{b^2 - 4ac}}{2a} - \frac{-b - \sqrt{b^2 - 4ac}}{2a}\right)^2 = \left(\frac{\sqrt{b^2-4ac}}{a}\right)^2 =$
    $\frac{b^2-4ac}{a^2}$
17. B $\frac{x^2}{9} - \frac{2}{3}x + 1 = 0$; $\left(\frac{x}{3} - 1\right)^2 = 0$; $\frac{x}{3} = 1$
18. A $y = \frac{\sqrt{(3x-5)(4x^2+12x+9)}}{6x^2 - x - 15}$
    $y = \frac{\sqrt{(3x-5)(2x-3)^2}}{(3x-5)(2x-3)} = \frac{(3x-5)^{\frac{1}{2}}(2x-3)}{(3x-5)(2x-3)}$
    $y = (3x-5)^{\frac{-1}{2}}$
19. E $\log_8(\sqrt{a+x} + \sqrt{a-x}) + \log_8(\sqrt{a+x} - \sqrt{a-x}) = \frac{1}{3}$
    $\log_8(a + x - a + x) = \frac{1}{3}$; $\log_8 2x = \frac{1}{3}$
    $2x = 8^{\frac{1}{3}}$ $x = 1$
20. D $x^{\frac{2}{3}} - 3x^{\frac{1}{3}} = 4$ $x^{\frac{2}{3}} - 3x^{\frac{1}{3}} - 4 = 0$
    $(x^{\frac{1}{3}} - 4)(x^{\frac{1}{3}} + 1) = 0$
    $x^{\frac{1}{3}} = 4$ $x=64$ $x^{\frac{1}{3}} = -1$ $x=-1$
    sum = 64 –1 = 63
21. B $\frac{a^x - a^{-x}}{2} = 3$ $a^x - \frac{1}{a^x} - 6 = 0$
    $a^{2x} - 6a^x - 1 = 0$ Solve using the quadratic formula:
    $a^x = 3 \pm \sqrt{10}$; $x=\log_a(3 + \sqrt{10})$
    remember that you can't take the log of a negative number

22. B Since x + y + z = 100 and x, y, and z are proportional to 2, 3, and 5 x = 20, y = 30, and z = 50 . y = ax – 10 30 = 20a – 10, a = 2

23. B If $\frac{m}{n}=\frac{4}{3}$ and $\frac{r}{t}=\frac{9}{14}$ then value of $\frac{3mr\text{-}nt}{4nt\text{-}7mr}$

$$m=\frac{4n}{3};\quad t=\frac{14r}{9}\qquad \frac{4nr-\frac{14rn}{9}}{\frac{4n(14r)}{9}-7\left(\frac{4n}{3}\right)r}$$

$$\frac{\frac{36nr-14nr}{9}}{\frac{56nr-84nr}{9}}=\frac{22nr}{9}\cdot\frac{9}{-28nr}=\frac{-11}{14}$$

24. C The base of the triangle is h + 2 and since the triangle is a 30-60-90 triangle h + 2 = $\sqrt{3}$ h

$(h+2)^2=(\sqrt{3}\ h)^2;\qquad h^2+4h+4=3h^2$

$h^2-2h-2=0$; solve using the quadratic formula $h=1\pm\sqrt{3}$ You can only use the positive one.

25. D $y=\frac{10^{\log x}}{x^3};\quad y=\frac{x}{x^3};\quad y=\frac{1}{x^2};\quad x^2y=1$

this is an inverse function

26. C Given $x^2+y^2-8x+2y-3=0$ the center is $(x-4)^2+(y+1)^2=20$ radius = $2\sqrt{5}$

C = $2\pi\left(2\sqrt{5}\right)=4\pi\sqrt{5}$

27. A Given the points (3,5) and (-2,1) the slope is

$m=\frac{4}{5}$, the $m_{\perp}=\frac{-5}{4}$. The midpoint between the given points is (.5, 3). The perpendicular bisector then is

–10x - 8y + 29 = 0

28. C Changing $x^2+4y^2-2x-24y-19=0$ into graphing form you get

$\frac{(x-1)^2}{56}+\frac{(y-3)^2}{14}=1$ the length of the semi-major axis is $\sqrt{56}=2\sqrt{14}$

the longest chord = $4\sqrt{14}$

29. C The y-intercept is when x = 0

$$\begin{vmatrix}1 & 0 & -1\\ 2 & y & 3\\ 3 & 2 & 1\end{vmatrix}=10\qquad y-4+3y-6=10$$

y = 5 (0, 5)

30. A Starting with xu = 400 substitute for u
x (v – 20) = 400, substituting for v you get

$x\left(\frac{400}{y}-20\right)=400$, substituting for y

$$x\left(\frac{400}{\frac{2x}{3}}-20\right)=400\ ;\qquad x\left(\frac{600}{x}-20\right)=400$$

600 – 20x = 400; x = 10

31. A

$$\left(x^2-9\right)^{\frac{3}{2}}=1\left(x^2\right)^{\frac{3}{2}}+\frac{3}{2}\left(x^2\right)^{\frac{1}{2}}(-9)^1+\frac{3}{8}\left(x^2\right)^{\frac{-1}{2}}(-9)^2+\ldots$$

The third term is $\frac{243}{8x}$

32. A Let b = rate of the boat in still water;
Let s = rate of the current. From the first trip we know that 5(b-s) = 2(b+s) which leads to 3b – 7s = 0. From the second trip we know 3(b+s) – 2 = 7(b-s) which leads to 2b - 5s + 1 = 0 Solving the system for s you will get s = 3.

33. C Given ΔABC, BC=1, AC=p, AB = $\sqrt{p^2+1}$

$\cos\angle A=\frac{AC}{AB}=\frac{p}{\sqrt{p^2+1}}$

34. C If $x=\sqrt{yz}$, then $x^2=yz$ and $y=\frac{x^2}{z}$.

Therefore, $\log y=\log\frac{x^2}{z}=2\log x-\log z$

35. B the teller's initial total could be represented by .25q + .1d + .05n + .01p. What he should have had could be represented by .25(q-x) + .1(d+x) + .05(n+x) + .01(p-x) = .25q +.1d + .05n +.01x -.25x + .1x + .05x -.01x = initial total -.11x

36. D

$$\begin{array}{c|ccc} \frac{-7}{2} & 3 & \frac{b}{2} & 18 \\ & & \frac{-21}{2} & \frac{-35}{2} \\ \hline & 3 & 5 & \frac{r}{2} \end{array} \qquad \frac{b}{2} - \frac{21}{2} = 5$$

and $18 - \frac{35}{2} = \frac{r}{2}$ then b = 31 and r = 1

r + b = 32

37. D $6^{a+b} = 6^2$ a+b = 2; $6^{a+5y} = 6^3$

a + 5b = 3; 4b = 1; b = .25 $a = \frac{7}{4}$

38. A A matrix is considered singular if its determinant is 0. Solving for the determinant x (1 + 15) – 2 (-30 – 2) = 0;

x = - 4 $B^2 = \begin{bmatrix} 3 & -1 \\ 1 & 4 \end{bmatrix}^2 = \begin{bmatrix} 8 & -7 \\ 7 & 15 \end{bmatrix}$

Q = - 4 R = 23; Q + R = 19

39. C 1 is false because the domain is all real numbers. 2 is false because if 0 < a < 1 f(4) < f(-1). 3, 4, and 5 are true.

40. D 2a + 2c = 32 ; a + c = 16; $a^2 + 8^2 = c^2$; $c^2 - a^2 = 64$; (c + a)(c – a) = 64; 16 (c – a) = 64; c – a = 4 solving the system – a + c = 4 and a + c = 16 ; c = 10 and a = 6. The area then is .5 (12)(8) = 48