How many edges comprise a K_{10}?

Discrete Math (Open Division) Solution Key

a. 100
b. 50
c. 45
d. 55

#1

1 3 6 10

Triangular numbers

$$\text{edges}(K_{10}) = \frac{9 \cdot 10}{2} = 45$$

$(p \wedge q) \rightarrow (p \vee q)$ is logically equivalent to

a. T
b. $p \vee q$
c. F
d. $p \wedge q$

#2

$$\begin{aligned}
(p \wedge q) \rightarrow (p \vee q) &\Longleftrightarrow \neg(p \wedge q) \vee (p \vee q) \\
&\Longleftrightarrow \neg p \vee \neg q \vee p \vee q \\
&\Longleftrightarrow (\neg p \vee p) \vee (\neg q \vee q) \\
&\Longleftrightarrow T \vee T \\
&\Longleftrightarrow T
\end{aligned}$$

How many relations are there on a set with n elements?

#3

a. $n!$
b. 2^n
c. n^2
d. 2^{n^2}

A relation on A is a subset of $A \times A$. $|A \times A| = n^2$ so there are 2^{n^2} relations on A.

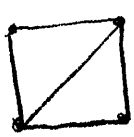
Which of the following graphs is planar?

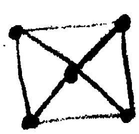
#4

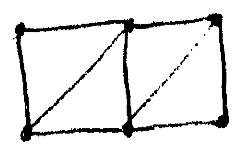
a. K_5
b. $K_{3,3}$
c. K_6
d. K_4

K_4 — no explanation necessary

5. Which graphs shown below have an Euler path?

I: 

II: 

III: 

a. I
b. I, II
c. I, III
d. II, III

I has 2 vert with odd degree
II has 4 vert with odd degree
III has 2 vert with odd degree

I, III have Euler paths.

6. Let R be a relation on $\mathbb{Z}^+$ (the positive integers) where a R b if a is a factor of b. ($a, b \in \mathbb{Z}^+$)
which of the following lists of properties best describes R?

a. symmetric, transitive
b. antisymmetric, transitive, reflexive
c. antisymmetric, symmetric, reflexive
d. symmetric, transitive, reflexive

aRa ← reflexive
If aRb and bRc then aRc ← transitive
1R2 but $2 \not R 1$ so not symmetric
If aRb and bRa, a = b ← antisymmetric

7. Let Q(x,y) denote "x-y = y+x". Which of the quantifications below are true?

I. $\exists x \exists y\, Q(x,y)$ II $\forall x\, Q(x,0)$ III $\exists x \forall y\, Q(x,y)$

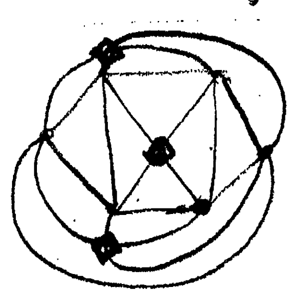
a. I
b. I, II
c. II, III
d. I, II, III

$\exists x \exists y\, Q(x,y) : T$ x = 1, y = 0 I, II
$\forall x\, Q(x,0) : T$ x = x
$\exists x \forall y\, Q(x,y) : F$

8\. The crossing number of a simple graph is the minimum number of crossings that can exist in a planar representation of the graph. A crossing is defined as two edges passing through one another.

What is the crossing number of K_6?

a. 0
b. 1
c. 2
d. 3



3

9\. Of the 25 possible values of a modulo 25 system, how many have modular inverses?

a. 10
b. 15
c. 20
d. 25

0, 5, 10, 15, 20 don't have modular inverses in mod 25 20

10\. The solution of the recurrence relation $H_n = 2H_{n-1} + 1$ when $H_1 = 2$ is ____

a. $H_n = 2^n - 1$
b. $H_n = 2^n$
c. $H_n = 2^n + 2^{n-1} - 1$
d. $H_n = 2^n + 2^{n-1} + 1$

$H_n = 2H_{n-1} + 1$
$= 2(H_{n-2} + 1) + 1$
$= 2^2(2H_{n-3} + 1) + 2 + 1$
$\vdots$
$= 2^{n-1}H_1 + 2^{n-2} + \dots + 2 + 1$
$= 2^n + 2^{n-2} + \dots + 2 + 1$
$= 2^n + 2^{n-1} - 1$

A

11. A string that contains only 0s, 1s, and 2s is called a ternary string. The recurrence relation and initial conditions for the number of ternary strings that do not contain two consecutive 0s is?

a. $A_n = A_{n-1} + A_{n-2}$, $A_1 = 3$, $A_0 = 1$
b. $A_n = 2A_{n-1} + A_{n-2}$, $A_1 = 3$, $A_0 = 1$
c. $A_n = 2A_{n-1} + 2A_{n-2}$ $A_1 = 3$, $A_0 = 1$
d. $A_n = A_{n-1} + 2A_{n-2}$, $A_1 = 3$, $A_0 = 1$

$A_0 = 1$ $\varnothing$
$A_1 = 3$ 0 1 2

To construct a strength of length n you either add a 1 or a 2 to a string of length $n-1$ or add 10 or 20 to a string of length $n-2$. $A_n = 2a_{n-1} + 2a_{n-2}$

$A_n = 2A_{n-1} + 2A_{n-2}$, $A_1 = 3$ $A_0 = 1$

12. What is the composite of the relation R and S where R is the relation from {1,2,3} to {1,2,3,4} with R = {(1,1), (1,4), (2,3), (3,1), (3,4)} and S is the relation from {1,2,3,4} to {0,1,2} with S = {(1,0), (2,0), (3,1) (3,2), (4,1)}?

a. {(1,0), (1,1), (2,1), (2,2), (3,0), (3,1)}
b. {(1,2), (2,2), (3,1), (3,2)}
c. {(1,0), (1,2), (3,1), (3,2)}
d. {(1,0), (1,1), (2,0), (3,1)}

a.

13. Assuming that no new n-tuples are added which fields are primary keys for the relation represented in the table below;

Professor	Department	Course #	Room	Time
Cruz	Zoology	335	A100	9:00 AM
Cruz	Zoology	412	A101	8:00 AM
Farber	Psychology	501	A100	3:00 PM
Grammer	Physics	617	A110	11:00 AM
ROSEN	CS	518	N521	2:00 PM
ROSEN	MATH	575	N502	3:00 PM

a. Professor
b. Course #
c. COURSE #, TIME
d. TIME

B

14. What is the symmetric closure of the relation $R = \{(a,b) \mid a > b\}$ on the set of positive integers?

a. $S = \{(a,b) \mid a < b\}$
b. $S = \{(a,b) \mid a \geq b\}$
c. $S = \{(a,b) \mid a = b\}$
d. $S = \{(a,b) \mid a \neq b\}$

D.

14

15. Find the number of path's of len 5 between two different vertices in K_4.

a. 60

b. 61

c. 62

d. 59

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}^5 = \begin{bmatrix} 60 & 61 & 61 & 61 \\ 61 & 60 & 61 & 61 \\ 61 & 61 & 60 & 61 \\ 61 & 61 & 61 & 60 \end{bmatrix}$$

16. What is the set of cut edges for the graph represented by the following adjancy matrix?

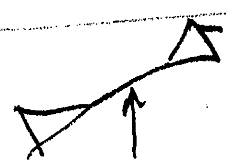
$$\begin{array}{c|c} & \begin{matrix} a & b & c & d & e & f \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \\ f \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

a. {ab, cd}

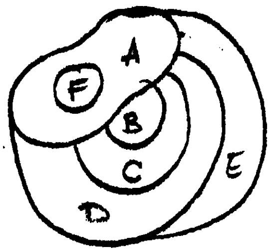
• b. {cd}

c. {cd, ef}

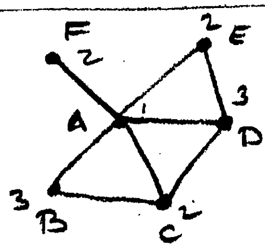
d {ef}



17.
Find the minimum number of colors needed to color the following map

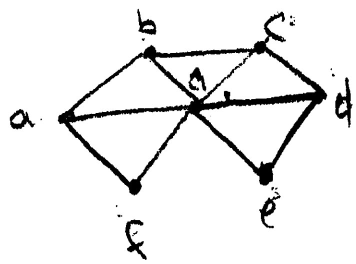


a. 1
b. 2
c. 3
d. 4



3 colors

19.
~~18.~~
What is the chromatic number of the following graph?



a. 1
b. 2
c. 3
d. 4

$\boxed{3}$

20. Suppose you are given three pitchers of water, of sizes 10 g, 7 g, and 4 g. Initially the 10 g pitcher is full and the other two empty. If a move is defined as pouring water from one pitcher to another, until the receiving pitcher is full or the pouring pitcher is empty. What is the smallest number of moves necessary to get 2 g in the 7-g pitcher?

a. 2
b. 3
c. 4
d. 5

10, 7, 4

$10,0,0 \xrightarrow{①} 6,0,4 \xrightarrow{②} 0,6,4 \xrightarrow{③}$

$4,6,0 \xrightarrow{④} 4,2,4$

4

18. Order the following functions in growth order from slowest to greatest.

I. $f(x) = e^x$ II. $f(x) = x$ III $f(x) = e^{.5 \ln x}$ IV $f(x) = x!$

a. II, III, I, IV
b. II, III, IV, I
c. III, II, IV, I
d. III, II, I, IV

$\sqrt{x} < x < e^x < x!$

d.

20

21.
Which of the following relations represented by materices are partial orders.

I. $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ II $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ III $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$

a. I, II, III
b. II, III
c. II
d. I, III

partial orderings are reflexive, transitive, anti symmeteic

I. r, a
II. r, a, t ←
III. r, a

22. A warelock goes to a store with one dollar to buy ingredients for his wife's Witch's Brew. The store sells bat tails for 5¢ apiece, lizard claws for 5¢ apiece, newt eyes for 5¢ apiece, and calf blood for 20¢ a pint bottle. How many different purchases (subsets) of ingredients will this dollar buy?

a 563
b 536
c 554
d 600

$$\binom{22}{2} + \binom{18}{2} + \binom{14}{2} + \binom{10}{2} + \binom{6}{2} + \binom{2}{2} = 536$$

Lets make this about integers

$.05T + .05S + .05E + .2B = 1$ becomes

$T+S+E+4B = 20$, or

$T+S+E = 20-4B$ an equation

with $C(20-4B+3-1,\ 20-4B) = C(22-4B,\ 2)$ for $B = 0,1,2,3,4,5$

23. A playoff between two teams consists of at most five games. The first team that wins three games wins the playoff. In how many different ways can the playoff occur?

a 18
b 15
c 20
d 21

USE A TREE DIAGRAM.

24. How many ways are there to select five bills from a money bag containing \$1 bills, \$2 bills, \$5 bills, \$10 bills, \$20 bills, \$50 bills, and \$100 bills? Assume that the orders in which the bills are chosen does not matter, that the bills of each denomination are indistinguishable, and that there are at least five bills of each type.

a 21
b 252
c 792
d 462

$C(7+5-1, 5) = 462$

25 How many one-to-one functions are there from a set with m elements to one with n elements ($m < n$)?

a $n \cdot m$
b n^m
c $P(n, m)$
d. $C(n, m)$

Another way to think of this problem is choosing the m end points of the mapping where order matters $P(n, m)$