(7)

$dv = -32dt$

$v = -32t + C_1 \quad t=0 \; v=1000$

$\Rightarrow C_1 = 1{,}000$

$v = \frac{ds}{dt} = -32t + 1000$

$ds = (-32t + 1000)\,dt$

$s = -\frac{32t^2}{2} + 1000t + C_2$

$s=0, t=0 \Rightarrow C_2 = 0$

$s = -16t^2 + 1000t$

$8000 = -16t^2 + 1000t$

$16t^2 + 1000t + 8000 = 0$

$t = \frac{250 \pm \sqrt{30{,}500}}{8}$

$t = 53.1$ or 9.4

$v = -32t + 1000$

$= -32(9.4) + 1000 \approx$

698.570 ft/sec

(11)

2nd order eq. $y'' + ay' + b = 0$

+ homogeneous

1st order $y' + Ky = 0$ has

standard soln. $y = Ce^{-Kx}$

so $y'' - y' - 2y = 0$ has

soln. of form $y = e^{mx}$

$y' = me^{mx}$

$y'' = m^2 e^{mx}$

$m^2 e^{mx} - me^{mx} - 2e^{mx} =$

$e^{mx}(m^2 - m - 2) = 0$

since $e^{mx} \neq 0$ $m^2 - m - 2 = 0$

$m = 2$ or $m = -1$

so 2 solutions

$y = e^{mx} \Rightarrow y = e^{2x}; \; y = e^{-x}$

so

$y = C_1 e^{2x} + C_2 e^{-x}$

(8)

$\frac{dx}{dt} = -kx \qquad x(0) = 1 \quad x(1) = \frac{3}{4}$

$x = Ce^{-Kt} \qquad C = 1$

$K = -\log\left(\frac{3}{4}\right) = .288$

$x = e^{-0.288t}$

$.99 = e^{-.288t}$

$t \approx .035$ inches thick

Vol rate of flow $= KA\sqrt{2gx}$

(13) $g = 32\ ft/s^2$

circular orifice $K = .6$

$\frac{dV}{dt} = -.6A\sqrt{64x}$

$A = \frac{\pi}{144}\ ft^2$ also $t = \frac{8}{\pi}$

$4\frac{dx}{dt} = \frac{dV}{dt} = \frac{-.6\pi}{144}\sqrt{64x}$

$4\frac{dx}{dt} = \frac{-\pi\sqrt{x}}{30}$

$\frac{dx}{\sqrt{x}} = \frac{-\pi}{120}\,dt$

$2x^{1/2} = -\frac{\pi}{120}t + c$

$t=0\ \ x=4 \Rightarrow c = 4$

$x^{1/2} = \frac{-\pi t}{240} + 2$ or $x = \left(\frac{480-\pi t}{240}\right)^2$

set $x = 0$

$t = \frac{480}{\pi}$ sec

Let u = Temp

(14) $\frac{du}{dt} = k(u-10)$ ← outside temp

$t=0,\ u=70°$

$t=3,\ u=25°$

$\int\frac{du}{u-10} = \int K\,dt$

$\ln|u-10| = Kt + c$

$u = Ce^{Kt} + 10$

$t=0\quad e^{Kt}=1$ so $70 = C+10$
$60 = C$

$t=3\quad 25 = 60e^{3K} + 10\quad K \simeq -.46$

$u = 10 + 60e^{-.46t}$

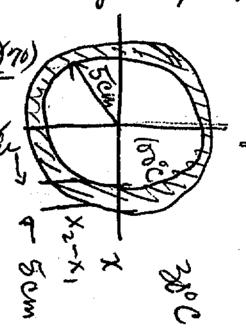
$t=7\quad u = 10 + 60e^{-.46(7)}$

(15) $K = 0.00060$ cal/cm, deg. sec., $L = 100$ cm

q(cal/sec) $= \frac{2\pi(0.00060)(100)(70)}{\ln 2}$

Change to hr ~~140,000~~ cal/hr

1,375,000 cal/hr



q = quantity of heat in body

A = area ⊥ to direction of flow = lateral surface of cylinder of radius x

L(cm) = length of pipe

$A = 2\pi x L$

$q = -KA\frac{du}{dx}$

$q = 2\pi KLx\frac{du}{dx}$

$q\int_5^{10}\frac{dx}{x} = -2\pi KL\int_{100}^{30}du$

$q\ln x\Big|_5^{10} = q\ln 2 = 2\pi KL(100-30)$

(21) $h(x) = \ln(g(x))$

$h''(x) = f(x) / (g(x))^2$

$h(x) = \ln(g(x))$

$h' = \frac{1}{g(x)} \cdot g'(x)$

$$h'' = \frac{g''(x)g(x) - g'(x)g'(x)}{[g(x)]^2}$$

$$= \frac{f(x)}{[g(x)]^2} \Longrightarrow$$

$$f(x) = g''(x)g(x) - [g'(x)]^2$$

$\boxed{\text{choice d}}$

(23) $\frac{dy}{dx} + 2y = x^2$

$(2y - x^2)\,dx + dy = 0$

$\frac{\partial M}{\partial y} = 2$ so int. fac $e^{\int +2dx} = e^{2x}$

$\underbrace{y'e^{2x} + 2e^{2x}y}_{d(ye^{2x})} = x^2e^{2x}$

so $ye^{2x} = \int x^2e^{2x}\,dx$

Int. by parts

$ye^{2x} = \frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C$

$y = \frac{1}{2}x^2 - \frac{1}{2}x + \frac{1}{4} + Ce^{-2x}$

$\boxed{\text{choice B}}$