Alpha Level - 2002 Probability Topic Test

1. ${}_6P_2 = 30$ $\boxed{C}$

2. $\dfrac{\binom{7}{3}\binom{5}{1}}{\binom{12}{4}} = \dfrac{175}{495} = \dfrac{35}{99}$ $\boxed{C}$

3. p(at least one received an A) = 1 - p(none received A)

$= 1 - \dfrac{\binom{35}{5}}{\binom{45}{5}} = 1 - .2657 = .7343$ C

4. $1 - [P(1) + P(2) + P(3) + P(4)]$

\# of balls selected until a green one is selected

$1 - \left[\frac{7}{15} + \frac{8}{15}\left(\frac{7}{14}\right) + \frac{8}{15}\cdot\frac{7}{14}\cdot\frac{7}{13} + \frac{8}{15}\cdot\frac{7}{14}\cdot\frac{6}{13}\cdot\frac{7}{12}\right]$

$1 - \left[\frac{7}{15} + \frac{4}{15} + \frac{28}{195} + \frac{14}{195}\right]$ $\boxed{A}$

$1 - .9487 \doteq .0512 = 2/39$

5. $\dfrac{2\cdot 7!}{8!} = \dfrac{2\cdot \cancel{7!}}{8\cdot \cancel{7!}} = \dfrac{1}{4}$ $\boxed{C}$

6. $\dfrac{\binom{8}{2}\binom{12}{3}}{\binom{20}{5}} = \dfrac{6160}{15504} = \dfrac{385}{969}$ $\boxed{B}$

7. $\dfrac{3!\,4!}{7!} = \dfrac{1}{35}$ $\boxed{C}$

8. P(at least one spade) = 1 - p(no spades)

$1 - \left[\frac{39}{52}\cdot\frac{38}{51}\cdot\frac{37}{50}\cdot\frac{36}{49}\right] \doteq .6962$ $\boxed{B}$

9. $\left(\frac{5}{6}\right)^6 = .335$ $\boxed{D}$

10. $\dfrac{\binom{5}{5}\binom{10}{5}}{\binom{15}{10}} = \dfrac{252}{3003} = \dfrac{12}{143}$ $\boxed{A}$

11. $\dfrac{5}{30} = \dfrac{1}{6}$ $\boxed{B}$

12. P(3 Red) ~~of~~ P(3 green)

$\dfrac{\binom{3}{3}}{\binom{9}{3}} + \dfrac{\binom{4}{3}}{\binom{9}{3}} = \dfrac{1}{84} + \dfrac{4}{84} = \dfrac{5}{84}$ $\boxed{B}$

13. $\dfrac{\binom{5}{2}\cdot\binom{21}{3}}{\binom{26}{5}} = \dfrac{13300}{65780} \doteq .2022$ $\boxed{B}$

14. P(Not SR and No 2nd Lang)

	Sr.	Jr.	Soph.	Freshmen
No 2nd lang	14		30	17
2nd lang.	6		5	3

$\frac{7}{9}$ of (Jr and Sr) don't take 2nd lang

$7/9\,(45) = 35$ don't take 2nd lang.

That means 21 juniors don't take a 2nd language

	Sr	Jr	Soph	Freshmen
No 2nd Lang	14	21	30	17
2nd lang	6	4	5	3

P(Not SR and No 2nd Lang) = P(Not SR) $\cdot$ P(No 2nd Lang / Not a Sr)

$\dfrac{80}{100} \cdot \left(\dfrac{68}{80}\right) = .68$ $\boxed{D}$

15. (Coin and Tails) · (Coin and Tails) or (Coin & Heads) · (Roll die get 4 or 5)

$\frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{2}{6} = \frac{1}{4} + \frac{1}{6} = \frac{3}{12} + \frac{2}{12} = 5/12$ B

16. P(vowel in 1st box) = 3/10 P(vowel in 2nd box = 2/16

P(vowel) = P(1st box is selected and then a vowel is selected)

or

P(2nd box is selected and then a vowel is selected).

$= \frac{1}{2}(3/10) + \frac{1}{2}(\frac{2}{16}) = \frac{3}{20} + \frac{1}{16} = \frac{12}{80} + \frac{5}{80} = \frac{17}{80}$ A

17. $P(W_T \mid W) = \frac{P(W_T \text{ and } W)}{P(W)} = \frac{\frac{1}{2}(4/6)}{\frac{1}{2}(4/6) + \frac{1}{2}(3/6)}$

If white is transferred, urn B now has 4 W, 2 B

If white is not transferred, urn B now has 3 W, 3 B

$= \frac{\frac{1}{3}}{\frac{1}{3} + \frac{1}{4}} = \frac{1/3}{7/12} = \frac{1}{3} \cdot \frac{\overset{4}{\cancel{12}}}{7} = \frac{4}{7}$ A

18. P(all 4 draw their own names) = $\frac{1}{4} \cdot \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{1} = \frac{1}{24}$

P(exactly 3 draw their own names) = 0 (4th would have to also)

P(exactly 2 draw their own names) = $\binom{4}{2} \frac{1}{4} \cdot \frac{1}{3} \cdot \frac{1}{2} = \frac{6}{24}$

P(exactly 1 draws his own name) = $\cancel{4}\left[\frac{1}{\cancel{4}} \cdot \frac{2}{3} \cdot \cancel{\frac{1}{2}}\right] = \frac{1}{3} = \frac{8}{24}$

P(exactly 0 draw their own name)

$= 1 - [1/24 + 0 + 6/24 + 8/24]$

$= 1 - [\frac{15}{24}] = 9/24 = \frac{3}{8}$ D

15. (Coin and Tails) · (Coin and Tails) or (Coin and Heads) · (Roll die get 4 or 5)

$\frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{2}{6} = \frac{1}{4} + \frac{1}{6} = \frac{3}{12} + \frac{2}{12} = 5/12$ B

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$= \frac{1}{2}(3/10) + \frac{1}{2}(\frac{2}{16}) = \frac{3}{20} + \frac{1}{16} = \frac{12}{80} + \frac{5}{80} = \frac{17}{80}$ A

17. $P(W_T \mid W) = \frac{P(W_T \text{ and } W)}{P(W)} = \frac{\frac{1}{2}(4/6)}{\frac{1}{2}(4/6) + \frac{1}{2}(3/6)}$

If white is transferred, urn B now has 4 W, 2 B

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$= \frac{\frac{1}{3}}{\frac{1}{3} + \frac{1}{4}} = \frac{1/3}{7/12} = \frac{1}{3} \cdot \frac{\cancel{12}^{4}}{7} = \frac{4}{7}$ A

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P(exactly 3 draw their own names) = 0 (4th would have to also)

P(exactly 2 draw their own names) = $\binom{4}{2} \frac{1}{4} \cdot \frac{1}{3} \cdot \frac{1}{2} = \frac{6}{24}$

P(exactly 1 draws his own name) = $\cancel{4}\left[\frac{1}{\cancel{4}} \cdot \frac{2}{3} \cdot \cancel{\frac{1}{2}}\right] = \frac{1}{3} = \frac{8}{24}$

P(exactly 0 draw their own name)

$= 1 - [1/24 + 0 + 6/24 + 8/24]$

$= 1 - [\frac{15}{24}] = 9/24 = \frac{3}{8}$ D

19. $P(B \mid \text{defective}) = \dfrac{P(B \text{ and defective})}{P(\text{defective})}$

$$= \frac{1/4 \cdot 3/100}{1/4(5/100) + \frac{1}{4}\left(\frac{3}{100}\right) + \frac{1}{4}\left(\frac{4}{100}\right) + 1/4(8/100)}$$

$$= \frac{\frac{3}{400}}{\frac{1}{4}\left(\frac{20}{100}\right)} = \frac{\frac{3}{400}}{\frac{1}{20}} = \frac{3}{\cancel{400}_{20}} \cdot \frac{\cancel{20}}{1} = \frac{3}{20} = .15 \quad \boxed{B}$$

20. $\frac{1}{2}$ $\boxed{A}$

21. $P(x \geq 11)$ where x is # of correct responses

$P(x \geq 11) = 1 - P(x \leq 10) = 1 - .9961 = .0039 \doteq .004$ $\boxed{B}$

(use TI-83 calculator "binom cdf"

22. ${}_3C_2 \left(\frac{5}{11}\right)^2 \cdot \left(\frac{6}{11}\right)^1 = 3\left(\frac{25}{121}\right)\left(\frac{6}{11}\right) = .338$ $\boxed{C}$

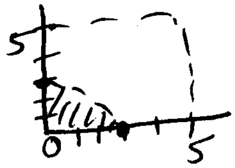
23. $\dfrac{11!}{2!\,2!\,2!\,2!}$ = total number of arrangements = 2,494,800

P(P will appear directly in front of I) = $\dfrac{10\left(1 \cdot 1 \cdot \frac{9!}{2!2!2!2!}\right)}{2{,}494{,}800}$

$$= \frac{\frac{10!}{16}}{2{,}494{,}800} = .0909 = \frac{1}{11} \quad \boxed{A}$$

24. P(2 sixes in 17 rolls) and another 6 on the next roll

$$\binom{17}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{15}\cdot\frac{1}{6} = \binom{17}{2}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^{15} = .041 \quad \boxed{C}$$

25. 

$$\frac{\text{area of successful region}}{\text{area of total region}} = \frac{\frac12(3)(3)}{25} = \frac{4.5}{25} = .18 \quad \boxed{D}$$

26. $$\frac{\text{area of successful region}}{\text{area of total region}} = \frac{\pi(3)^2}{18^2} = \frac{9\pi}{(18)^2} = \frac{\pi}{36} \quad \boxed{A}$$

89
1461

27. $P(1\ \text{Rep}) + P(3\ \text{Rep})$

$= P(1R, 3D) + P(3R, 1D)$

$$\frac{{}_{15}C_1 \cdot {}_{24}C_3}{{}_{39}C_4} + \frac{{}_{15}C_3 \cdot {}_{24}C_1}{{}_{39}C_4} \quad \boxed{A}$$

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$.369 + .133 = .502$

28.
Round I
$P(H) = 1/2$
$P(A \text{ wins}) = \frac12\left(\frac12\right) = 1/4 = P(TH)$
$P(B \text{ wins}) = P(TTH) = 1/8$
$P(C \text{ wins})$

Round II
$P(A \text{ wins}) = TTTH = 1/16$
$P(B \text{ wins}) = TTTTH = 1/32$
$P(C) \text{ wins} = TTTTTH = 1/64$

Round III
$P(6T, 1H) = 1/128$
$P(7T, 1H) = \frac{1}{256}$
$P(8T, 1H) = \frac{1}{512}$

$P(B \text{ wins}) = 1/4 + 1/32 + \frac{1}{256} + \ldots$

Infinite geometric series

$$S = \frac{a_1}{1-r} = \frac{\frac14}{1-\frac18} = \frac{\frac14}{7/8} = \frac14\cdot\frac87 = \boxed{\frac{2}{7}} \quad \boxed{D}$$

29. Every 4 years – there is one Feb 29

$P(\text{Feb 29 B'day}) = \frac{1}{1461}$

$P(\text{at least one}) = 1 - P(\text{none}) > .5$

$1 - \left(\frac{1460}{1461}\right)^n > .5$

$.5 > \left(\frac{1460}{1461}\right)^n$

$(\ln .5) > n \ln\left(\frac{1460}{1461}\right) \quad \boxed{B}$

$1012.3 < n \quad 1013$

30. $P(\text{total} = 2 \text{ or } 3 \text{ or } 5 \text{ or } 7 \text{ or } 11)$

$$\frac{1}{36} + \frac{2}{36} + \frac{4}{36} + \frac{6}{36} + \frac{2}{36} = \frac{15}{36} = \frac{5}{12} \quad 5/12 \quad \boxed{D}$$

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