Inner School Test Part A

This test can be worked on by any student member of your delegation. No sponsor or adult is to assist with any part of this test. One answer sheet must be turned in by designated time by each school. Ties will be broken based upon reasoning demonstrated in neat and legible work attached to the answer sheet.

1. Which American president devised an original proof of the Pythagorean Theorem?
2. Although Blaise Pascal is a very famous mathematician, his father is also a good mathematician. Who is Blaise's father and what curve is named after him?
3. The letter A stands for a digit. Find all possible values of A if A1AA2 is divisible by 36.
4. Match the symbol with the person who is credited with their first use.

1. $>,<$	a. Thomas Harriot
2. $+,-$	b. Johann Heinrich Ralin
3. $\times$	c. William Oughtred
4. $\div$	d. Robert Recorde
5. $=$	e. Johann Widman

5. What is the last digit in the sum $1!+2!+3!+\cdots+99!$?
6. Explain the rule for the following number arrangement.

8	5	4	9	1	7	6	3	2	0

7. Starting at the top and moving down how many ways can you spell ALGEBRA?

$$A$$
$$L \quad L$$
$$G \quad G \quad G$$
$$E \quad E \quad E \quad E$$
$$B \quad B \quad B$$
$$R \quad R$$
$$A$$

8. You are a designer for a logo company. An order is placed for a logo having 7 equilateral triangles and 9 edges. Sketch the logo.
9. If $f(x) = x^3 + x^2 - x$, find all points where f(x) and its inverse intersect.

10. It is an odd number with three digits. All the digits are different and add up to 12. The difference between the first two digits equals the difference between the last two digits. What is the number?

11. A barber's pole is 2 m in height and has a radius of 20 cm. If a red stripe is painted on the pole so that it starts at the bottom, ends at the top, and wraps around the pole 8 times, then how long is the stripe?

12. If 4 toothpicks are needed to form one square, 12 toothpicks are needed to form 4 congruent squares, and 24 toothpicks are needed to form 9 congruent squares, then find the minimum number of toothpicks needed to form 199 congruent squares?

13. The sum of "n" distinct positive integers is 78. Find the greatest possible value of "n"?

14. Complete the "Cross Number" puzzle:

1	2	3
4		
5		

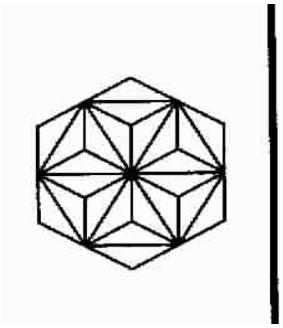
Across
1. A friendly number
4. A perfect square
5. A different friendly number

Down
1. A palindrome
2. $\frac{1}{2}$ of 556
3. A perfect square

15. Find the remainder for the quotient $2^{1000} \div 25$.
16. A polygon has 1000 sides. Find the number of diagonals that can be drawn in the polygon.

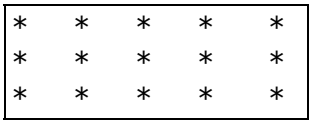
17. A bird hops along the top of a moving train. From the front of the train to the back the bird hops 60 times. From the back of the train to the front, the bird hops 210 times. If the bird hops once each second, then how fast does the train move?

18. The poet Longfellow, in his novel *Kavanagh*, introduced several clever mathematical problems from an ancient Sanskrit work. One follows... "If one fifth of a bee hive of bees flew to the ladamba flower, on third flew to the slandbara, three times the difference of these two numbers flew to an arbor, and one bee continued to fly about attracted on each side by fragrant ketaki and the malati, what was the number of bees?



19. This tessellation consists of congruent obtuse triangles arranged in a regular hexagon. What is the number of isosceles triangles of any size in the figure?

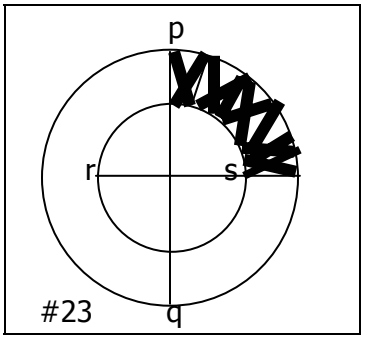
20. What is the probability that four different points chosen at random from the fifteen equally spaced points shown are the vertices of a square? Express answer as a common fraction.



21. Two ferry boats start moving at the same instant from opposite sides of the Hudson River, one boat going from New York to Jersey City, and the other going from Jersey City to new York. One boat is faster than the other so they meet at a point 720 yards from the shore. After arriving at their destination, each boat remains 10 minutes in the slip to change passengers; then it begins the return trip. The boats again meet at a point 400 yards from the other shore. What is the exact width of the river?

22. Where do the following two graphs intersect? What is the area of the region inside the cardioid and outside the circle?
$r = 4 + 4\cos\theta$ *and* $r = 10\cos\theta$.

23. Given the figure and the facts that $pq \perp rs,\ pq = 5mm,\ rs = 3mm$, find the area of the shaded region.



24. What mathematician is the word algorithm named after?
25. Solve over $[0, 2\pi)$: $\sin x + \sin 2x + \sin 3x = 0$

26. A car repair shop knows from experience when someone calls to say a car will not start that 70% of the time the engine is flooded, 30% of the time the battery is weak, and 20% of the time neither one of these problems exists. What is the probability that a person calling to say their car will not start has a flooded engine and a weak battery?
27. A town has two competing evening local news shows: one on channel 7 and one on channel 11. The people who watch the news every evening are divided into two groups: those who watch channel 7 and those who watch channel 11. Seventy percent of those who watch channel 7 one night watch channel 7 the next night, the remaining 30% watch channel 11 the next night. Twenty percent of those who watch channel 11 one night watch channel 7 the next night, the remaining 80% watch channel 11 the next night. Suppose a survey shows that 60% of the people who watch the news every night watched the channel 7 news last night and 40% watched channel 11. What percentage of the regular news watchers watch channel 7 and what percentage watch channel 11 tomorrow night?
28. Delta and United both plan to add another city to their list of cities with nonstop flights from Boston, Minneapolis, Philadelphia, and St. Louis seem to be the best choice for each airline. If both airlines add Minneapolis to their list, then Delta will show a gain of $75,000 over United. If both add Philadelphia, Delta will show a gain of $50,000 over United. However, if Delta chooses Philadelphia and United chooses with of the other two, Delta will show a profit of $200,000 over United. If Delta chooses Minneapolis and United chooses St. Louis or vice versa, then they are even. Finally, if United chooses Philadelphia, and Delta chooses Minneapolis, then Delta has a gain of $25,000 over United, but if Delta chooses St. Louis, United has a gain of $50,000 over Delta. Which city should each airline choose?
29. Ten arrows are shot at the target. One of them misses the target completely. The others all strike it. If the total sum of points is one hundred, in which part of the target did each arrow strike?
30. Determine the rotation equations that will transform the equation $4x^2 - 3xy + 5y^2 - 2x = 0$ into an equation without a mixed-product term.

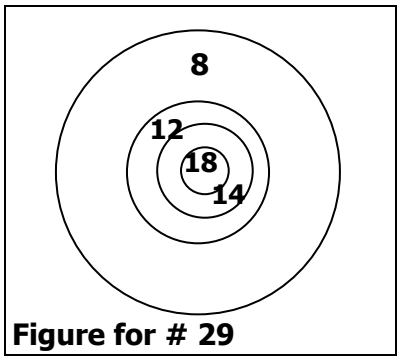
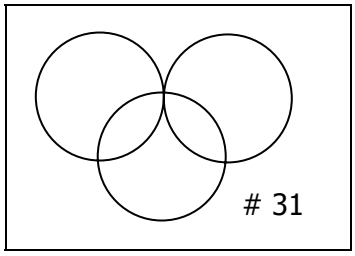


Figure for # 29

31. Three go-cart tracks are built as shown. Each track forms a separate one-thir of a mile loop. Three go-carts begin at the same time from the central point where all three tracks cross. One go-cart travels at 6 mph, another at 12 mph, and the third at 15 mph. How long will it take for all 3 go-carts to cross paths for the 5th time?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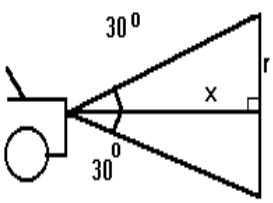
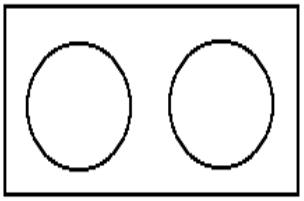
32. Alf, Bert and Cash are suspects in a robbery case. Their trial reveals the following facts: Either Cash is innocent or Bert is guilty. If Bert is guilty then Cash is innocent. Alf and Cash never work together and Alf never does a job on his own. Also, if Bert is guilty, so is Alf. Who is guilty?
33. Five members of a basketball team are weighed and an average weight is recalculated after each member is weighed. If the average increases by one kilogram each time, how much heavier is the last player than the first one? What if the difference is always two kilograms?
34. A set of consecutive positive integers beginning with 1 is written on the teacher's blackboard. A student came along and erased one number. The average of the remaining numbers is 35 7/17. What number was erased?

35. A car's headlights make two circular bright spots on a wall when the car is 10 m away. At what rate is the area of the bright spots changing when the car is 6 m away. The velocity of the car is 3 m/s.

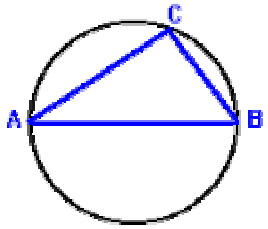




36. A chemical company spends x million dollars on research and finds that its profit can be expressed as a function of the amount of money spent on research. If $P(x) = 30 + 6 \log (x+2)$ how much will the company have to spend on research to increase its profit from its present level, with a research investment of 5 million dollars (P(5)), to 80 million dollars.

37. Two cars race around a circular track, in opposite directions, at constant rates. They start at the same point and meet every 30 seconds. If they move in the same direction, they meet every 120 seconds. If the track is 1800 m long, what is the speed of each car in km/hr?

38. In the diagram, AB is the diameter of the circle. If AB is 10 cm and the area of the triangle is 11 cm^2, find the perimeter of triangle ABC.



39. A van travels a maximum of 100 km/h. Its speed decreases in proportion with the number of passengers. The van can carry a maximum of seven people. Given that the van can travel 88 km/h with 3 people in the van, what will be the speed of the van when 6 people are on board?

40. A car traveling at 40 m/s (144 km/h) passes a police car hidden behind a billboard. Two seconds after the car passes the billboard the police car is on the road accelerating at 3.0 m/s squared. How long will it take the police to catch the speeder.

Tie Breaker: What is the value of each letter? $\frac{I}{DO} = 0.SITSITSIT\ldots$