

Inner School Test Part A answers pg 1

1. James Garfield

2. Etienne Pascal "Limacon of Pascal"

3. $A = 5$

4. (1) Harriett (a)
 (2) Widman (e)
 (3) Oughtred (b)
 (4) Ralin (c)
 (5) Recorde (d)

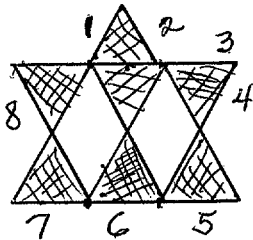
5. 3 Observe pattern

$$\begin{aligned} 1! &= 1 \\ 1! + 2! &= 3 \\ 1! + 2! + 3! &= 9 \\ 1! + \dots + 4! &= 33 \\ 1! + \dots + 5! &= 153 \\ &\vdots \end{aligned}$$

6. Alphabetical Order

7. 20

8.



7 triangles; 9 edges

9-10 typed

11. Unwind stripe, S ; $S = \frac{2\sqrt{25 + 64\pi^2}}{5}$
 ≈ 10.250 cm

12. 220

13. 12

14.

2	2	0
6	7	6
2	8	4

Inner School Test Answers pg. 2

25. $\sin x + \sin 2x + \sin 3x = 0$

$\sin x + 2\sin x \cos x + \sin(2x+x) = 0$

$\sin x + 2\sin x \cos x + 3\sin x \cos^2 x - \sin^3 x = 0$

$\sin x = 0 \quad 1 + 2\cos x + 3\cos^2 x - \sin^2 x = 0$

$1 + 2\cos x + 3\cos^2 x - 1 + \cos^2 x = 0$

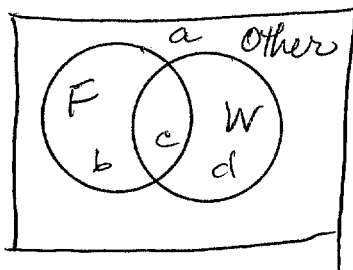
$2\cos x + 4\cos^2 x = 0$

$2\cos x (1 + 2\cos x) = 0$

$\cos x = 0 \quad \cos x = -\frac{1}{2}$

$\left\{ 0, \pi, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3} \right\}$

26.



F = flooded

W = Weak battery

$P(F) = .7 = b + c$

$P(W) = .3 = c + d$

$P(\text{Other}) = .2 = a$

$P(F \cap W) = c$

$P(F \cup W) = .8 = b + c + d$

$b = .5$

$d = .1$

so $c = .2$

20%

or
.2

27. 55%

ch. 7 ch. 11

Distribution $[\begin{smallmatrix} .6 & .4 \end{smallmatrix}]$ Transition $\begin{matrix} c7 & c11 \\ c7 & \begin{bmatrix} .7 & .3 \\ .2 & .8 \end{bmatrix} \\ c11 & \end{matrix}$

$[\begin{smallmatrix} .5 & .5 \end{smallmatrix}] [T] = [\begin{smallmatrix} .45 & .55 \end{smallmatrix}]$

28. Each should choose Philadelphia

Payoff matrix

		United		
		P	M	SL
Delta	P	50,000	200,000	2,000,000
	M	25,000	25,000	0
	SL	-50,000	0	-100,000

Inner School

Tiebreaker

$$\begin{array}{l} I=6 \quad S=1 \\ D=3 \quad T=2 \\ O=7 \end{array} \left. \vphantom{\begin{array}{l} I=6 \\ D=3 \\ O=7 \end{array}} \right\} \frac{6}{37} = .162162 \dots \quad \text{Tiebreaker}$$

Realizing that $\frac{SIT}{999} = .SITSIT\dots$

Consider reducing 999 by a factor that would become a 2-digit number. $27 \times 37 = 999$ choose 37

Now test single digit numerators

$$\text{Then } \frac{I}{37} = .SITSIT\dots$$

Inner School Test Part A: Answers & Selected solutions

(Numbers not listed here have hand-written scanned solutions.)

9. Wherever the function $f(x)$ intersects the line $y = x$, it also intersects its inverse. Following this logic through gives us the following solutions **(0,0), (-2,-2), (1,1)**.

10. Answer: 741

$$abc \quad 100a + 10b + c$$

$$a + b + c = 12$$

$$a - b = b - c$$

$$a > b + c$$

$$a + b + c = 12$$

$$-a + 2b -/+ c = 0/3b = 12$$

$$b = 4$$

$$a + c = 8$$

$$8 - c > 4 + c$$

$$4 > 2c$$

$$c < 2$$

abc is odd numbered so the digit "c" must be odd.

Hence $c = 1$, $a = 7$ and $b = 4$ NUMBER 741

15. 16

16. 498,500

17. 1.8 (velocity of the bird)

18. 15

19. 36

20. $\frac{2}{195}$

21. 1760 yds

22.

23. π

24. Al- Khowarizmi

29. 2 at 8 points and 7 at 12 points

30.
$$x = \frac{1}{2}\sqrt{2-\sqrt{2}} x' - \frac{1}{2}\sqrt{2+\sqrt{2}} y'$$

$$y = \frac{1}{2}\sqrt{2+\sqrt{2}} x' + \frac{1}{2}\sqrt{2-\sqrt{2}} y'$$

31. $33\frac{1}{3}\text{min or } \frac{5}{9}\text{hr}$

32. Alf and Bert are guilty. If Bert is innocent, then Cash is innocent and Alf is guilty; but Alf never works alone. Therefore, no one is guilty. Therefore, Bert cannot be innocent; he must be guilty. If Bert is guilty, then Cash is innocent and Alf is guilty.

33. 8, 16

Player Number	Weight	Cumulative Average
1	x	x
2	x+2	x+1
3	(x+1)+3	x+2
4	(x+2)+4	x+3
5	(x+3)+5	x+4

Fifth player weighs 8 more kilograms than the first!
 When difference is 2 lbs ->5th player weighs 16 more kilograms.

34. 7

Let n be the last number on the board. The largest possible average is obtained if the digit 1 is erased; the average is then

$$(2+3+\dots+n)/(n-1) = ((n+1)n/2 - 1)/(n-1) = (n+2)/2$$

The smallest average possible is obtained when n is erased the average then:

$$n(n-1)/2(n-1) = n/2 \quad 1+2+\dots+n-1/n-1$$

Thus

$$n/2 \leq 35 \frac{7}{17} \leq n+2/2$$

$$n \leq 70 \frac{14}{17} \leq n+2$$

$$68 \frac{14}{17} \leq n \leq 70 \frac{14}{17}$$

Therefore

n = 69 or 70. Since $35 \frac{7}{17}$ is the average of (n-1) integers $(35 \frac{7}{17})(n-1)$ must be an integer and n is 69.

If x is the number erased.

$$1/2(69)(70) - x/68 = 35 \frac{7}{17}$$

$$69 \frac{14}{17} - x = 35 \frac{7}{17} \quad 68$$

$$x = 7$$

35. $12\pi \frac{m^2}{\text{sec}}$

$$A = \pi r^2$$

$$= \pi x^2 / 3$$

$$r = x \tan 30 \text{ degrees} = x / 3^{.5}$$

$$dx / dt = 3 \text{ m/s}$$

$$\begin{aligned} dA / dt &= dA / dx * dx / dt \\ &= (2\pi * x / 3) * 3m/s \\ &= 2\pi * x m/s \end{aligned} \quad \text{at } x = 6 \quad \begin{aligned} dA/dt &= 2 \pi (6) m^2/s \\ &= 12 \pi m^2/s \end{aligned}$$

36. \$215.54 million

$$\begin{aligned} P(5) &= 30 + 6 \log(5 + 2) / \log 2 \\ &= 46.84 \text{ million dollars} \\ 80 &= 30 + 6 \log_2 (x + 2) \\ 50 / 6 &= \log_2 (x + 2) \\ 2^{(50/6)} &= (x + 2) \\ x &= 320.54 \text{ million dollars.} \end{aligned}$$

therefore: increase in spending = 320.54 - 5 = 215.54 million

37. $c_1 = 81 \text{ km/hr}$ $c_2 = 135 \text{ km/hr}$

Let $x =$ rate of car 1 (m/s)
 Let $y =$ rate of car 2 (m/s)

$$\begin{aligned} 1800m &= 30x + 30y \\ 1800 - 30x &= 30y \\ 7200 - 120x &= 120y \\ 1800 + 120x &= 120y \end{aligned}$$

Solve for Car 1:

$$\begin{aligned} 7200 - 120x &= 1800 + 120x \\ 5400 &= 240x \\ x &= 22.5 \text{ m/s} \end{aligned}$$

Convert to km/hr

$$x = 22.5 \text{ m/sec} * 60 \text{ sec/min} * 60 \text{ min/hr} = 81 \text{ km/hr}$$

1000 m/km

Solve for Car 2:

Substitute $x = 22.5 \text{ m/s}$ $1800 + 120x = 120y$
 $y = 37.5 \text{ m/s}$

Convert to km/hr

$$x = 37.5 \text{ m/sec} * 60 \text{ sec/min} * 60 \text{ min/hr} = 135 \text{ km/hr}$$

1000 m/km

38. 21.998

$$0.5 * b * h = 11$$

$$\begin{aligned} b &= \text{length AC} \\ b * h &= 22 \\ b &= 22/h \end{aligned}$$

$$\begin{aligned} h &= \text{length CB} \\ c &= \text{length AB} \end{aligned}$$

$$\begin{aligned} c^2 &= b^2 + h^2 \\ 100 &= b^2 + h^2 \\ 100 &= (22/h)^2 + h^2 \\ 100 &= 22^2/h^2 + h^2 \\ 100h^2 &= 22^2 + h^4 \\ h^4 - 100h^2 + 284 &= 0 \end{aligned}$$

To solve, use quadratic formula:

$$h^2 = \frac{100 + \sqrt{(100)^2 - 4(484)}}{2} = 9.74$$

$$h^2 = \frac{100 - [(100)^2 - 4(484)]}{2} = 2.258$$

Since $b = 22/h$, then $b = 9.74$ <----- Therefore, use $h = 2.258$

39. 76 km/hr

$$100 - 3t = 88$$

$$100 - 88 = 3t$$

$$t = 12/3$$

$$t = 4 \text{ km/h per person}$$

When six persons are on board, the van travels at

$$100 - 6t = 100 - 6(4) = 76 \text{ km/h}$$

40. 15.1 sec

$t_0 = 2s$ for speeder

$ds = 40 \text{ m/s to } + 40 \text{ m/s } t$

$dp = 3.0 \text{ m/s}^2 (t)^2$

$ds = dp$

$$40(2) + 40t = 3.0t^2$$

$$3.0t^2 - 40t - 80 = 0$$

$$t = 40 \pm \sqrt{(1600 + 4(3)(80))}^{.5} = 15.1, -1.77 \quad \text{time always } (t)$$

It will take the police 15.1 s to catch the speeder.