

Inner School Test Part A: Answers & Selected solutions

(Numbers not listed here have hand-written scanned solutions.)

9. Wherever the function $f(x)$ intersects the line $y = x$, it also intersects its inverse. Following this logic through gives us the following solutions **(0,0), (-2,-2), (1,1)**.

10. Answer: 741

$$abc = 100a + 10b + c$$

$$a + b + c = 12$$

$$a - b = b - c$$

$$a > b + c$$

$$a + b + c = 12$$

$$-a + 2b -/+ c = 0/3b = 12$$

$$b = 4$$

$$a + c = 8$$

$$8 - c > 4 + c$$

$$4 > 2c$$

$$c < 2$$

abc is odd numbered so the digit "c" must be odd.

Hence $c = 1$, $a = 7$ and $b = 4$ NUMBER 741

15. 16

16. 498,500

17. 1.8 (velocity of the bird)

18. 15

19. 36

20. $\frac{2}{195}$

21. 1760 yds

22.

23. π

24. Al- Khwarizmi

29. 2 at 8 points and 7 at 12 points

30.
$$x = \frac{1}{2}\sqrt{2-\sqrt{2}} x' - \frac{1}{2}\sqrt{2+\sqrt{2}} y'$$

$$y = \frac{1}{2}\sqrt{2+\sqrt{2}} x' + \frac{1}{2}\sqrt{2-\sqrt{2}} y'$$

31. $33\frac{1}{3} \text{ min } \text{ or } \frac{5}{9} \text{ hr}$

32. Alf and Bert are guilty. If Bert is innocent, then Cash is innocent and Alf is guilty; but Alf never works alone. Therefore, no one is guilty. Therefore, Bert cannot be innocent; he must be guilty. If Bert is guilty, then Cash is innocent and Alf is guilty.

33. 8, 16

Player Number	Weight	Cumulative Average
1	x	x
2	x+2	x+1
3	(x+1)+3	x+2
4	(x+2)+4	x+3
5	(x+3)+5	x+4

Fifth player weighs 8 more kilograms than the first!
 When difference is 2 lbs -> 5th player weighs 16 more kilograms.

34. 7

Let n be the last number on the board. The largest possible average is obtained of the digit 1 is erased; the average is then

$$(2+3+\dots+n)/(n-1) = ((n+1)n/2 - 1)/(n-1) = (n+2)/2$$

The smallest average possible is obtained when n is erased the average then:

$$n(n-1)/2(n-1) = n/2 \quad 1+2+\dots+n-1/n-1$$

Thus

$$n/2 \leq 35 \frac{7}{17} \leq n+2/2$$

$$n \leq 70 \frac{14}{17} \leq n+2$$

$$68 \frac{14}{17} \leq n \leq 70 \frac{14}{17}$$

Therefore

n = 69 or 70. Since $35 \frac{7}{17}$ is the average of (n-1) integers $(35 \frac{7}{17})(n-1)$ must be an integer and n is 69.

If x is the number erased.

$$1/2(69)(70) - x/68 = 35 \frac{7}{17}$$

$$69 \cdot 35 - x = 35 \cdot (7/17) \cdot 68$$

$$x = 7$$

35. $12\pi \frac{m^2}{\text{sec}}$

$$A = \pi r^2$$

$$= \pi x^2 / 3$$

$$r = x \tan 30 \text{ degrees} = x / 3^{.5}$$

$$dx / dt = 3 \text{ m/s}$$

$$h^2 = \frac{100 - [(100)^2 - 4(484)]}{2} = 2.258$$

Since $b = 22/h$, then $b = 9.74$ <----- Therefore, use $h = 2.258$

39. 76 km/hr

$$100 - 3t = 88$$

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$$t = 12/3$$

$$t = 4 \text{ km/h per person}$$

When six persons are on board, the van travels at

$$100 - 6t = 100 - 6(4) = 76 \text{ km/h}$$

40. 15.1 sec

$$t_0 = 2 \text{ s for speeder}$$

$$ds = 40 \text{ m/s to } + 40 \text{ m/s } t$$

$$dp = 3.0 \text{ m/s}^2 (t)^2$$

$$ds = dp$$

$$40(2) + 40t = 3.0t^2$$

$$3.0t^2 - 40t - 80 = 0$$

$$t = 40 \pm \sqrt{(1600 + 4(3)(80))}^{.5} = 15.1, -1.77 \quad \text{time always } (t)$$

It will take the police 15.1 s to catch the speeder.