2002 National Mu Alpha Theta Convention
Alpha Level—Individual Test

1. Which of the following cannot be expressed as the difference of the squares of two nonconsecutive integers?
A. 43 B. 44 C. 45 D. 48 E. All of these

2. If p and q are distinct primes each of which is greater than 3, let *d* denote the number of positive integers n such that p^13*q^5 divided by n is a positive integer. Which of the following is *d*?

A. 84 B. 65 C. 48 D.110 E. NOTA

3. The sum of the solutions to 1000x^2-500x+73737=0 is :
A. 1/2 B. 10 C. -1/2 D. 1000 E. NOTA

4. A debating team with 4 members is to be chosen from among 20 students. Find the number of distinct possible teams. (Of course two teams are considered the same if they have the same members, even if the members were chosen in different orders).

A. 116280 B). 2000 C. 3000 D. 4845 E. 5000

5. What is the sum of all three solutions to x^3-x^2-4x-6=0.

A) 2 B) -1 C) 3 D) -3 E) NOTA

6. What is the magnitude of the difference between 105311^2 and 105305^2.

A) 1263696 B) 1243692 C) 210622 D) 210610 E) NOTA

7. Two marbles are drawn from a vase which contains 4 white marbles, 6 blue marbles, and 8 green marbles. What is the probability, if I pick two marbles at random, they will be the same color?

A) $\frac{49}{153}$ B) $\frac{58}{153}$ C) $\frac{15}{153}$ D) $\frac{53}{156}$ E) NOTA

8. If x and y satisfy the equation y=(300-x)(x+100) then the largest possible value of y is closest to
A) 5000 B)10,000 C) 25,000 D) 50,000 E) 100,000

9. Find the remainder when $x^5 + 4x^4 - 3x^2 + 1$ is divided by $x^2 - 1$.

A) $x - 1$ B) -1 C) 3x D) x+2 E) NOTA

10. The coefficient of x^50 in (x+1)^100 is

A) $\frac{100!}{51!49!}$ B) $\frac{100!}{(50!)^2}$ C) 101 D) 1189 E) NOTA

11. Find the sum of the values for the solutions for x, y, and z:

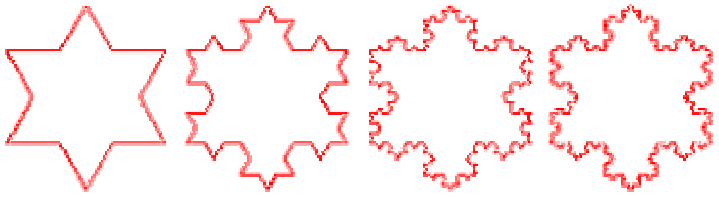
x+2y+6z=10
3x+4y+2z=12
2x+3y+3z=6

A) 2 B) 16 C) 8 D) -4 E) NOTA

12. Given an 8x8 chess board, a rectangle is considered admissible if each of its 4 distinct corners is a vertex on the chess board and if one of its sides is parallel to the bottom of the chess board. Squares are to be treated as rectangles. Which of the following is the number of admissible rectangles contained in the chess board?

A) 200 B) 404 C) 1296 D) 816 E) NOTA

13. To construct a Koch Snowflake, first you draw an Equilateral Triangle. Then you trisect each of the sides, and construct on the middle segment another equilateral triangle. If this is repeated infinitely, the resulting shape will be a Koch Snowflake (see picture). If a Koch Snowflake is created on an equilateral triangle with sides of length 2, what is its area?



A) $4\pi/3$ B) $\frac{11\sqrt{3}}{5}$ C) $\frac{6\sqrt{3}}{5}$ D) $2\sqrt{3}$ E) NOTA

14. Evaluate $\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{3+\ldots}}}}$.

A) $\sqrt{15}$ B) $\frac{1}{6}$ C) $\frac{\sqrt{15}-3}{2}$ D) $\frac{\sqrt{15}+3}{2}$ E) NOTA

15. If (x1,y1) and (x2,y2) are solutions to (x-23)^2+(y-37)^2=1000 then the largest possible value of the distance between (x1,y1) and (x2,y2) is closest to

A) 20 B) 40 C) 60 D) 80 E) 100

16. If the point (0,y) is on the line passing through (20,30) and (50,100) then y is:

A) -50/3 B) 230/3 C) 650/3 D) -10 E) NOTA

17. 1+2+3…+999+1000 =

A) 50,000 B) 100,000 C) 500,500 D) 1,500,000 E) 5,000,000

18. Harry, Burly, and Joe are to receive a combined total of five pennies from Mr. Wilson's will. Assuming each person receives at least zero pennies and no more than five pennies, the number of possible ways in which the treasure can be divided is closest to which number?

A) 10 B) 20 C) 30 D) 40 E) 50

19. Ten balls, numbered 1 through 10, are placed in a bag. Draw one ball at random from the bag, put it back, and then draw a second time. What is the probability that 4 divides the product of your two selections?

A) ½ B) ¼ C) 9/20 D) 13/24 E) NOTA

20. An equilateral triangle is inscribed in a circle which is then inscribed in a square. What is the ratio of the area of the triangle to the area of the square?

A) $\frac{\sqrt{3}}{4}$ B) $\frac{3\sqrt{3}}{16}$ C) $\frac{5\sqrt{3}}{16}$ D) $\frac{3\sqrt{3}}{10}$ E) NOTA

21. How many 5 digit palindromes are there? (Palindromes are numbers which read the same forwards and backwards).

A) 1000 B) 100 C) 2187 D) 900 E) NOTA

22. Suppose a sequence is defined as follows. $a_1 = 2, \quad a_{n+1} = \frac{1}{3 - a_n}$

It can be shown that the sequence is decreasing and $0 \le a_n \le 2$ for all n.

Find the limit of the sequence.

A) $\frac{3+\sqrt{5}}{2}$ B) $\frac{1}{3}$ C) $\frac{3-\sqrt{5}}{2}$ D) 1 E) NOTA

23. How many positive integers are there whose digits are all different?

A) 623,529 B) 9,864,100 C) 986,409 D) 8,877,690
E) NOTA

24. A ball is dropped from a height of 100 feet onto a hard level surface. Suppose that each time it bounces, it rebounds to half of its previous height. If the ball continues to bounce indefinitely, find the total distance that it travels.

A) 200 ft B) 300 ft C) 150 feet D) 400 ft E) NOTA

25. Suppose a bookstore purchases 10 copies of a book at \$6.00 each and sells them at \$12.00 each with the understanding that at the end of a 3-month period any unsold copies can be redeemed for \$2.00 each. Find the net revenue of the bookstore owner at the end of the three-month period.

A) R(x) = 10x-40 B) R(x) = 12x-60 C) R(x) = 10X+20
D) R(x) = 12x –2 E) NOTA

26. For what value of the constant *c* will there be no inverse for the matrix

$$\begin{bmatrix} 1 & 4 & c \\ 2 & -1 & 7 \\ 3 & -2 & 11 \end{bmatrix}$$

A) -2 B) -1 C) 0 D) 1 E) 2

27. Let $x = \sqrt{2+\sqrt{2}} - \sqrt{2-\sqrt{2}}$. Then $384x^2 - x^8 =$

A) 442 B) 444 C) 448 D) 452 E) NOTA

28. For what positive constant *c* will the function $f(x) = x^2 + cx + 2$ have minimum value of -1 ?

A) 2 B) 6 C) $\sqrt{6}$ D) $3\sqrt{2}$ E) $2\sqrt{3}$

29. If $x = \log_3 5$ and $y = \log_3 7$, then $\log_3 \frac{45}{7} =$

A) $\frac{2x}{y}$ B) $\frac{3x}{y}$ C) $\frac{x+2}{y}$ D) x-y+2 E) 2(x-y)

30. Suppose (x,y) is a solution to the system: $\left\{\begin{matrix} xy+y^2=12 \\ x^2-xy=8 \end{matrix}\right\}$

Find the product of all possible values for y.

A) 4 B) 24 C) 48 D) 72 E) 96

31. Find $\cot\left[\frac{1}{2}Arc\sin\left(\frac{-3}{5}\right)\right]$

A) -3 B) 3 C) 1/3 D) -1/3 E) NOTA

32. The trigonometric form of $2i$ is :

A) $2\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$ *B*) $2\left(\cos\pi+i\sin\pi\right)$

C) $2\left(\sin\frac{3\pi}{2}+i\cos\frac{3\pi}{2}\right)$ D) $-2(\cos\pi+i\sin\pi)$ E) NOTA

33) Compute the ratio of the area to the perimeter of a triangle with sides measuring 10, 12, and 14.

A) $\frac{2\sqrt{6}}{3}$ B) $24\sqrt{2}$ C) $3\sqrt{3}$ D) $2\sqrt{6}$ E) NOTA

34) To which of the following is $\frac{(x+y)^3(z^2-1)}{(x^3+y^3)(z^4-1)(x+y+z)}$ equivalent?

A) $\frac{x^2y^2}{z^2+1}$ B) $\frac{x^2+y^2}{z^2+1}$ C) $((x^2-xy+y^2)(z^2+1)(x+y+z))^{-1}$

D) $\frac{x^2+y^2}{(x+y+z)^2}$ E) NOTA

35) How many six digit numbers (leading digit cannot be zero) are there such that any two adjacent digits a have a difference of no more than one? Consider that 0 can be adjacent to 9.

A) 2187 B) 2430 C) 320 D) 288 E) NOTA

36) A circle is inscribed in a square which is inscribed in another circle. What is the ratio of the area of the inner circle to the outer circle?

A) 1:2 B) 1:3 C) 1:4 D) $\sqrt{2}:2$ E) NOTA

37) Find the sum of all real x such that $\left|x^2-3\right|=\left|3x+1\right|$.

A) -2 B) -1 C) 0 D) 3 E) NOTA

38) If $f(x)=x^3+3x-5$ and $f(g(x))=x^5-3x^4+x^2-1$ then what is g(2)?

A) -2 B) -1 C) 0 D) 1 E) NOTA

39) 1024 people participate in a chess tournament in which each player plays all the others exactly once. How many games are played?

A) 523776 B) 575986 C) 361840 D) 610053 E) NOTA

40) Three married couples, one of which is Ellen and Michael Pearson, have purchased theatre seats and are seated in a row consisting of just six seats. If they sit completely randomly, what is the probability that Ellen and Michael are seated next to each other?

A) 2/3 B) 1/15 C) 1/3 D) 1/5 E) NOTA

41) The wages of three men for four weeks is $108. At the same rate of pay, how many weeks will 5 men work for $135?

A) 2.5 B) 3 C) 3.5 D) 4 E) 4.5

42) Two boys paddled a canoe 6 miles down a river to the point where the river flowed into a lake; then they paddled 4 miles across the lake to a fishing ground. Later they returned over the same route to their starting point. If the trip to the fishing ground required 2 hours, the return trip required 4 hours, and there was no current in the lake, find the rate of the current (mph) in the river?

A) 5/4 B) 2 C) 5/2 D) 4 E) NOTA

43) If $x^3 + cx + d = 0$ has a pair of equal roots, find $4c^3 + 27d^2$.

A) -5 B) 0 C) 23 D) 31 E) NOTA

44) Solve the inequality: $\begin{vmatrix} x & x-2 \\ 5 & 10 \end{vmatrix} < 0$

A) x < -3 B) x < -2 C) x>2 D) x>3 E) NOTA

45) ABCD is a trapezoid with AB the lower base. Angle A measures 90 degrees, Angle B measures 60 degrees, and BC = CD = 8. Find the area of the trapezoid.

A) 20 B) $20\sqrt{3}$ C) 40 D) $40\sqrt{3}$ E) 60

46) Find the measure of an angle that is equal to 4/5 of its supplement.

A) 45 degrees B) 60 degrees C) 75 degrees D) 80 degrees
E) NOTA

47) Compute the value of x if $5x - 1 = 3 + \cfrac{2}{1 + \cfrac{2}{1 + \cfrac{2}{\ddots}}}$

A) 1 B) -2, 1 C) 2/5 D) 6/5 E) NOTA

48) The mass of a spherical body varies jointly as its density and the cube of its radius. Find the ratio of the mass of Jupiter to that of Earth if the density of Jupiter is 5/22 that of Earth and its radius is 11 times that of Earth.

A) 8/125 B) 5/29282 C) 5/2 D) 605/2 E) NOTA

49) A train moving at r mph can cover a given distance in h hours. By how many mph must its speed be increased to cover the same distance in one hour less time?

A) $\frac{r}{h}$ B) $\frac{r}{h+1}$ C) $\frac{r}{h-1}$ D) $\frac{r+1}{h}$ E) $\frac{r-1}{h}$

50) Let $f(t)=1+a^t$. Find $\frac{1}{f(t)}+\frac{1}{f(-t)}$ in lowest terms.

A) 0 B) 1 C) $\frac{2}{1-a^{2t}}$ D) $\frac{2}{1-a^{t^2}}$ E) NOTA

Alpha—Individual Test

1. A
2. A
3. A
4. D
5. E
6. A
7. A
8. D
9. D
10. B
11. B
12. C
13. E
14. C
15. C
16. A
17. C
18. B
19. C
20. B
21. D
22. C
23. D
24. B
25. A
26. B
27. C
28. E
29. D
30. D
31. A
32. A
33. A
34. E
35. A
36. A
37. C
38. E
39. A
40. C
41. B
42. B
43. B
44. B
45. D
46. D
47. A
48. D
49. C
50. B