

Mu Alpha Theta National Convention: Denver, 2001
Real and Complex Numbers Topic Test – Theta Division

1. Simplify: $(4 - i\sqrt{3}) - (3i - 2)$
(A) $6 - (3 + \sqrt{3})i$ (B) $6 - 3i\sqrt{3}$ (C) $2 - (3 + \sqrt{3})i$ (D) $2 - (3 - \sqrt{3})i$ (E) NOTA

2. Find the sum of the following complex numbers: $4 + 3i$, $2i - 1$, $4i + 7$, $2 - 6i$, $6 + i$
(A) $18 + 4i$ (B) $10 - 6i$ (C) $23 + 10i$ (D) $-2 + 10i$ (E) NOTA

3. Evaluate: $(5 - 4i)(i + 4)$
(A) $18 + 12i$ (B) $-4 + 23i$ (C) $24 - 11i$ (D) $9 + 5i$ (E) NOTA

4. Evaluate: $(4 - 3i\sqrt{3})(2 + 6i\sqrt{3})$
(A) $-16\sqrt{3} + 18i\sqrt{3}$ (B) $48\sqrt{3} + 12i$
(C) $62 + 18i\sqrt{3}$ (D) $16\sqrt{3} + 18i\sqrt{3}$ (E) NOTA

5. Evaluate: $\frac{3 + 2i}{4 + 2i}$
(A) $1 + i$ (B) $\frac{8 + i}{10}$ (C) $1 - 2i$ (D) $i + 2$ (E) NOTA

6. Solve for b : $5 + \frac{b}{2 + i\sqrt{2}} - \frac{b}{2 - i\sqrt{2}} = 0$
(A) $4 + i$ (B) $3i\sqrt{2}$ (C) 4 (D) $-\frac{15i\sqrt{2}}{2}$ (E) NOTA

7. Determine the sum of A and B in the system of equations.
$$2A + B = 3i - 8$$
$$3A + 2B = 5 + 4i$$

(A) $5i$ (B) $\frac{2 + 3i}{3}$ (C) $-4 + 5i$ (D) $13 + i$ (E) NOTA

8. How many complex roots does $f(x) = 3x^3 - 15x - 9$ have?
(A) 0 (B) 1 (C) 2 (D) 3 (E) NOTA

Mu Alpha Theta National Convention: Denver, 2001
Real and Complex Numbers Topic Test – Theta Division

9. What is the sum of the roots of $f(x) = 2x^2 + 6x - 53$?
- (A) -4 (B) -3 (C) 4 (D) 6 (E) NOTA
10. What is the product of the roots of $f(x) = 2x^2 - 3x + 4$?
- (A) 1 (B) $\frac{3}{2}$ (C) 2 (D) 3 (E) NOTA
11. What is the product of the roots of $y = x^3 + 2x^2 - 3x + 4$?
- (A) -4 (B) -3 (C) 3 (D) 4 (E) NOTA
12. What is the sum of the reciprocals of the roots of $y = 2x^2 + 5x + 18$?
- (A) -2 (B) $-\frac{9}{10}$ (C) $-\frac{5}{18}$ (D) $\frac{5}{2}$ (E) NOTA
13. What is the magnitude of $6 - 8i$?
- (A) -2 (B) 8 (C) $6 + 8i$ (D) 10 (E) NOTA
14. What is the measure, in degrees, of the smaller angle in the complex plane between the x-axis and the position vector for the point $-2 + 2i\sqrt{3}$?
- (A) 90° (B) 120° (C) 135° (D) 150° (E) NOTA
15. Which of the following is equal to $2e^{\frac{3\pi i}{2}}$?
- (A) $-2i$ (B) $2 \operatorname{cis} 30^\circ$ (C) $i - 1$ (D) $\operatorname{cis} 270^\circ$ (E) NOTA
16. Which of the following is equal to $\frac{2e^{\frac{3\pi i}{2}} \times e^{\frac{2\pi i}{3}}}{3e^{3\pi i}}$?
- (A) $\frac{2e^{-\frac{5\pi i}{6}}}{3}$ (B) $\frac{e^{-\frac{2\pi i}{3}}}{3}$ (C) $\frac{2e^{\frac{\pi i}{2}}}{3}$ (D) $e^{-\frac{2\pi i}{3}}$ (E) NOTA

Mu Alpha Theta National Convention: Denver, 2001
Real and Complex Numbers Topic Test – Theta Division

17. Evaluate: $(1+i)^9$
- (A) $16-16i$ (B) $-16-16i$ (C) $-16+16i$ (D) $16+16i$ (E) NOTA
18. Which of the following are subsets of the imaginary numbers?
- I. The real numbers
II. The complex numbers
III. The irrational numbers
- (A) I only (B) II only (C) I & III only (D) II & III only (E) NOTA
19. Which of the following are complex numbers?
- I. 4
II. i
III. $4+i$
IV. $4-i$
- (A) I & II only (B) III & IV only
(C) II, III, & IV only (D) I, II, III, & IV (E) NOTA
20. Which of the following is a quadratic equation with roots $4 \pm i\sqrt{2}$?
- I. $x^2 - 4x + 13 = 0$
II. $x^2 - 8x + 18 = 0$
III. $2x^2 + 8x - 3 = 0$
- (A) I only (B) II only (C) III only (D) I & III only (E) NOTA
21. The equation $2x^2 + kx - m = 0$, where k and m are integers, has a root at $3 + ki$. What is the value of k ?
- (A) 153 (B) $\sqrt{153}$ (C) $2m$ (D) -12 (E) NOTA
22. What is the sum of the real roots of $2x^3 + 3x^2 - 4x + 15 = 0$?
- (A) -3 (B) $\frac{3}{2}$ (C) $-\frac{3}{2}$ (D) 2 (E) NOTA
23. A cubic equation with integral coefficients has $6 + 8i$ as one of its roots, has a leading coefficient of 1, and the coefficient of the quadratic term is 10. What is the value of the constant term?
- (A) 2200 (B) 1440 (C) -200 (D) -400 (E) NOTA

Mu Alpha Theta National Convention: Denver, 2001
Real and Complex Numbers Topic Test – Theta Division

24. Which of the following are square roots of $-i$?

- I. $1-i$
- II. $1+i$
- III. $i-1$
- IV. $-1-i$

- (A) I & III only (B) II & IV only
(C) I, II, & III only (D) I, II, III, & IV (E) NOTA

25. Which of the following are complex sixth roots of 1?

- I. $\text{cis } 60^\circ$
- II. $\text{cis } 180^\circ$
- III. $\text{cis } 270^\circ$

- (A) I only (B) I & II only (C) I & III only (D) I, II, & III (E) NOTA

26. Which of the following statements are true?

- I. Multiplication of complex numbers is commutative.
- II. The rational numbers are a subset of the complex numbers.
- III. The multiplicative inverse of a complex number z is equal to the complement of z divided by the magnitude of z .

- (A) II only (B) I & II only (C) II & III only (D) I, II, & III (E) NOTA

27. What is the sum of the five complex fifth roots of $2+i$?

- (A) 0 (B) 1 (C) $2+i$ (D) $2-i$ (E) NOTA

28. What is the product of the six complex sixth roots of $i-12$?

- (A) 1 (B) $12+i$ (C) $12-i$ (D) $i-12$ (E) NOTA

29. Which of the following is not equal to $3e^{\frac{4\pi}{3}}$?

- (A) $\frac{3}{2} + \frac{3\sqrt{3}}{2}i$ (B) $3 \text{ cis } 120^\circ$ (C) $3e^{\frac{2\pi}{3}}$ (D) $3e^{\frac{8\pi}{3}}$ (E) NOTA

30. Consider two complex numbers, $Z_1 = a+bi$ (where a and b are non-zero real numbers) and $Z_2 = -1-i$. By how many degrees must the vector from the origin to Z_1 in the complex plane be rotated (counter-clockwise) to be aligned with the vector from the origin to Z_1Z_2 ?

- (A) 45° (B) 150° (C) 225° (D) 270° (E) NOTA

Mu Alpha Theta National Convention: Denver, 2001
Real and Complex Numbers Topic Test – Theta Division

31. Z_1 and Z_2 are n th roots of 1. Which of the following is not necessarily an n th root of 1?

- (A) $Z_2 + Z_1$ (B) $Z_1 Z_2$ (C) $\frac{Z_1}{Z_2^3}$ (D) 1 (E) NOTA

32. A quartic equation with integer coefficients has $3i + 1$ and $4 - i$ as two of its roots. Given that the leading coefficient is 1, what is the coefficient of the quadratic term?

- (A) -12 (B) -10 (C) -6 (D) 16 (E) NOTA

33. Which of the following are roots of $x^3 - cx + k = 0$, given that $3 + 6i$ is a root and c and k are elements of the integers?

- I. -6
II. 6
III. $6i - 3$
IV. $3 - 6i$

- (A) IV only (B) I & IV only (C) II & IV only (D) III & IV only (E) NOTA

34. Evaluate: $\sum_{n=1}^6 (2i)^n$

- (A) $110 + 34i$ (B) $28 - 44i$ (C) $-52 + 26i$ (D) $-40 + 28i$ (E) NOTA

35. Evaluate: $\sum_{n=1}^{143} (i)^n$

- (A) 1 (B) i (C) -1 (D) $-i$ (E) NOTA

36. $Z_1 Z_2 = 4 + 2i$ and $\frac{Z_2}{Z_1} = \frac{11 + 2i}{5}$. Determine all possible values of Z_2 .

- (A) $Z_2 \in \{3 + i, 3 - i\}$ (B) $Z_2 \in \{3 + i\}$
(C) $Z_2 \in \{3 + i, -3 - i\}$ (D) $Z_2 \in \left\{3 + i, \frac{3 - i}{5}\right\}$ (E) NOTA

37. A teacher placed an equation of the form $x^2 + bx + c = 0$ on the board to be solved. Joe miscopied the value of c and got $5 \pm 2i$ as the roots. Jim miscopied the value of b , resulting in roots of $2 \pm i$. Julie copied the problem down correctly... What will she get for roots?

- (A) $5 \pm 2\sqrt{5}$ (B) $2 \pm 2i\sqrt{5}$ (C) $2 \pm i\sqrt{5}$ (D) $5 \pm 2i\sqrt{5}$ (E) NOTA

Mu Alpha Theta National Convention: Denver, 2001
Real and Complex Numbers Topic Test – Theta Division

38. Given that $2+i$ is a root of $x^3 - rx^2 - 7x - k = 0$, where r and k are integers, which of the following are also roots of the equation?

- I. $2-i$
- II. $i-2$
- III. -3

(A) I only (B) II only (C) I & III only (D) II & III only (E) NOTA

39. Consider the equation $z^2 + (4-2i)z - 8i = 0$ where z is a complex number. Let A be the root of greater magnitude, and B be the other root. Determine $A - B$.

(A) $4i$ (B) $-4-2i$ (C) $7-3i$ (D) $5+8i$ (E) NOTA

40. A sixth degree polynomial with a leading coefficient of 1 and rational coefficients has $8-2i$ and $3+i$ as roots, as well as a coefficient of 12 for its x^5 term. What is the largest possible value for the constant term?

(A) 196,520 (B) 204,000 (C) 220,320 (D) 224,400 (E) NOTA