

MU TRIGONOMETRY TOPIC TEST 2001 NATIONAL CONVENTION

① $56^\circ \cdot \frac{\pi}{180^\circ} = \frac{14\pi}{45}$ (A)

② $\frac{43\pi}{18} \cdot \frac{180^\circ}{\pi} = 430^\circ$ (D)

③ $\sin 0 - \cos 30^\circ + \tan \frac{\pi}{4} - \sec 60^\circ + \csc \frac{\pi}{2} - \cot 110^\circ = 0 - \frac{\sqrt{3}}{2} + 1 - 2 + 1 - (-\frac{1}{\sqrt{3}}) = -\frac{\sqrt{3}}{6}$ (D)

④ $1 + \tan^2 57^\circ = \sec^2 57^\circ = \csc^2 (90^\circ - 57^\circ) = \csc^2 33^\circ$ (C)

⑤ $\sin(\cos \frac{\pi}{2}) = \sin 0 = 0$ (D)

⑥ $2(\sin^2 y \cos y + \cos^2 y \sin y) = 2 \sin y \cos y (\sin^2 y + \cos^2 y) = \sin 2y$ (B)

⑦ $(\cos 73^\circ \cos 13^\circ + \sin 73^\circ \sin 13^\circ) + (\sin 37^\circ \cos 8^\circ + \cos 37^\circ \sin 8^\circ) = \cos(73^\circ - 13^\circ) + \sin(37^\circ + 8^\circ) = \frac{1}{2} + \frac{\sqrt{3}}{2}$ (A)

⑧ $\frac{1}{\cot w} + \frac{1}{\tan w} = \frac{\tan w + \cot w}{\tan w \cdot \cot w} = \tan w + \cot w = \frac{143}{64} \Rightarrow \text{area} = 212$ (B)

⑨ Law of sines $\frac{BC}{\sin A} = \frac{AB}{\sin C} \therefore BC = \frac{\sin A}{\sin C} AB = \frac{\sin 79^\circ}{\sin 36^\circ} = \frac{\sin 101^\circ}{\cos 34^\circ}$ (B)


⑩ $f'(0) = -\csc \theta \cot \theta = -(\frac{25}{24})(-\frac{7}{24}) = \frac{175}{576}$ (C)

⑪ $K = \frac{1}{2} ab \sin C$. Therefore, our desired ratio (since, a, b stay constant) is $\frac{\sin 2C}{\sin C} = 2 \cos C = 2(\frac{55}{73}) = \frac{110}{73}$ (C)


⑫ Period is $\frac{2\pi}{6} = \frac{\pi}{3}$ (D)

⑬ $\cos(-u) + \sin(-v) \cos(u) = \cos u - \sin v \cos u = \frac{5}{13} - \frac{4}{5} \cdot \frac{5}{13} = \frac{1}{13}$ (B)

⑭ The radius of the cross-section is $\sqrt{25 - 25 \cos^2 37^\circ} = 5 \sin 37^\circ$, so the area is $25\pi \sin^2 37^\circ = 25\pi (\frac{1 - \cos 74^\circ}{2}) =$ (A)

⑮  $\tan \theta = \frac{x}{75000} \therefore \sec^2 \theta \frac{d\theta}{dt} = \frac{1}{75000} \frac{dx}{dt} \therefore \text{③ } x = 37,500, \sec^2 \theta = \frac{5}{4} \therefore \frac{d\theta}{dt} = \frac{4}{5} \cdot \frac{1}{75000} \cdot 16500 = \frac{22}{125}$ (C)

⑯ $2(1 - \cos^2 B) - \cos B = 1 \Rightarrow 2\cos^2 B + \cos B - 1 = 0 \Rightarrow (2\cos B - 1)(\cos B + 1) = 0 \therefore \cos B = \frac{1}{2}$ or $\cos B = -1 \therefore B = \pi/3, \pi, 5\pi/3$ (D)

⑰  $x = b \cos A$ $y = a \cos B$ $\therefore b \cos A + a \cos B = x + y = AB = 50000$ (C)

(18) $y = \sin^{-3} x \frac{\cos^3 x}{\sin^3 x} \cos^2 x \frac{1}{\cos^5 x} = 1 \therefore y' = 0$ (B)


(19) $(\sin \theta)(1 + \tan \theta) = 1 + \tan \theta \therefore \sin \theta = 1$ or $1 + \tan \theta = 0$.
When $\sin \theta = 1$, $\tan \theta$ undef, so $\frac{7}{2}$ is not a sol'n. $\tan \theta = -1$ yields 37% , 7% (D)

(20) $\cos \angle BCD = \cos \angle CAB = \frac{5}{13}$; $\sec \angle ACD = \sec \angle CBA = \frac{13}{12}$ $\frac{5}{13} + \frac{13}{12} = \frac{229}{156}$ (A)

(21) $\frac{1473}{23} = \approx 64.043$ $\frac{773\pi}{12} \approx 64.418\pi$; x is always in 1st quadrant, so $\sin x$ is strictly increasing (C)

(22) $y' = \frac{10}{3} \cos \frac{5x}{3} - \frac{15}{4} \sin \frac{3x}{4}$, which has amplitude $\frac{10}{3} + \frac{15}{4} = \frac{85}{12}$ (C)
(Max attained @ $x = 18\pi$) (Find max by looking for x st. $\frac{5x}{3} = 2n\pi$ & $\frac{3x}{4} = 2m\pi + \frac{3\pi}{2}$)

(23) $\tan(\arctan \frac{22}{7}) = \tan(\arctan \frac{3}{4} + \arctan \frac{17}{x}) = \frac{\frac{20}{x}}{1 - \frac{51}{x^2}} = \frac{22}{7} \Rightarrow 11x^2 - 70x - 11(6) = 0$.
 $x = 11, -\frac{51}{11} \therefore$ Clearly $x > 0$, so $x = 11$ (B)

(24)  We seek $\frac{dx}{dt}$ (our answer is $2 \frac{dx}{dt}$) since there are two sides equally moving away from each other. $\cos \theta = \frac{x}{10} \therefore -\sin \theta \frac{d\theta}{dt} = \frac{1}{10} \frac{dx}{dt}$.
 $\frac{dx}{dt} = -10\sqrt{2}$ @ $\theta = 45^\circ$. $2 \frac{dx}{dt} = 20\sqrt{2}$ (B)

(25) $u = \cos t \therefore \frac{du}{dt} = -\sin t \therefore dt = \frac{du}{-\sin t} \therefore \int \sin^2 t \cos^2 t dt = -\int (1-u^2) u^2 du =$
 $\frac{u^3}{3} - \frac{u^5}{5} + C = \frac{\cos^3 t}{3} - \frac{\cos^5 t}{5} + C$ (C)

(26) $\cos A = -\frac{45}{53}$, $\sin B = -\frac{20}{29} \therefore \cos(A+B) = (\frac{-45}{53})(\frac{21}{29}) - (-\frac{20}{29})(-\frac{20}{29}) = -\frac{1505}{1537}$ (B)

(27) $V = \int_0^{\pi/4} \pi [(\cos x)^2 - (\sin x)^2] dx = \pi \int_0^{\pi/4} (\cos x - \sin x) dx = \pi [\sin x + \cos x]_0^{\pi/4} = \pi(\sqrt{2}-1)$ (D)


(28) Only $(\sin x, \cos x)$ of the $\binom{6}{2} = 15$ possibilities satisfy the criterion (B)

(29) $y = 3\sin x - 4\sin^3 x = \sin x (3 - 4\sin^2 x) = 0$ at $x=0$, $x=\pi/3$, and $y > 0$ for $0 < x < \pi/3$.
Hence, sought area is $A = \int_0^{\pi/3} (3 - 4\sin^2 x) \sin x dx$ let $u = \cos x$; $\frac{du}{dx} = -\sin x$.
 $A = \int_1^{\frac{1}{2}} (3 - 4(1-u^2)) \sin x dx = \int_1^{\frac{1}{2}} (1-4u^2) du = \left[u - \frac{4u^3}{3} \right]_1^{\frac{1}{2}} = \frac{2}{3}$ (B)

(30) $\sin^2 5\varphi = 1 \Rightarrow 5\varphi = 2m\pi + \pi/2 \Rightarrow \varphi = \frac{2m\pi}{5} + \pi/10$
 $5\varphi = 2m\pi + 3\pi/2 \Rightarrow \varphi = \frac{2m\pi}{5} + 3\pi/10$

The first satisfies $\frac{47\pi}{50} < \varphi < \frac{8\pi}{5}$ for $m=3$, the second for $m=2, 3$.

The sum of the three sol'n's is $39\pi/10$ (B)

31  $y = x + 25^\circ$ $\frac{\sin x}{150} = \frac{\sin 25^\circ}{200} \therefore x = 18.47^\circ \therefore y \approx 43^\circ$ (D)

32 $x = \sin u$; $\frac{dx}{du} = \cos u \Rightarrow dx = du \cos u \Rightarrow \int \frac{dx}{\sqrt{1-x^2}} = \int \frac{du \cos u}{\sin^2 u \cos u} = \int \frac{du}{\sin^2 u} = -\cot u + C$
 $\sqrt{1-x^2} = \cos u \therefore -\cot u + C = -\frac{\sqrt{1-x^2}}{x} + C$ (C)

33 $\cos x - \sin 2x - \cos 3x = \cos x - 2\sin x \cos x - \cos 2x \cos x + \sin 2x \sin x =$
 $\cos x (1 - 2\sin x - \cos 2x + 2\sin^2 x) = \cos x (1 - 2\sin x - 1 + 2\sin^2 x + 2\sin^2 x) =$
 $\cos x (4\sin^2 x - 2\sin x) = 2\cos x (\sin x) (2\sin x - 1) = 0$ where $\cos x = 0, \sin x = 0, \sin x = \frac{1}{2}$
 \therefore Solutions are $0, \pi/2, \pi, 3\pi/2, 2\pi, \pi/6, 5\pi/6$ (B)

34 Period of $2\sin 3x$ is $2\pi/3$, 0° to $\tan \frac{5x}{4}$ is $4\pi/5$, Thus the period is the 'LCM' of these, of 4π , (B)

35 $\sin \beta = \frac{2}{3}$ $\sin 3\beta = \sin 2\beta \cos \beta + \cos 2\beta \sin \beta = 2\sin \beta \cos^2 \beta + (2\cos^2 \beta - 1)\sin \beta$
 $= 2(\frac{2}{3})(\frac{\sqrt{5}}{3}) + \frac{2}{3}(\frac{+2-3}{3}) = \frac{11\sqrt{5}}{12}$ (B)

36 $\ln y = x \ln \sin x \Rightarrow \frac{1}{y} \frac{dy}{dx} = \ln \sin x + x \frac{\cos x}{\sin x} \Rightarrow \frac{dy}{dx} = (\sin x)^x (\ln \sin x + \frac{\cos x}{\sin x})$ (A)

37 $K = \frac{1}{2}(14)(12) \sin 15^\circ = 7 \cdot 12 (\frac{\sqrt{5}-1}{4}) = 21\sqrt{5} - 21 \therefore a+b = 26$ (A)

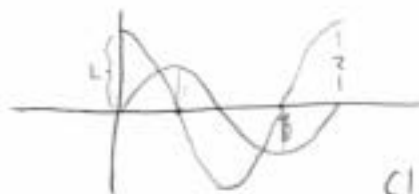
38 Only $\sin x$ & $\cos x$ satisfy it (C)

39 $\cos(8x) = \cos(4x)$ when $8x = 2\pi - 4x$ & $4x = 2\pi + 4x$, giving solution $x = 2\pi/7$
 $8x = 4\pi - 4x, 4x = 4\pi + 4x$, gives $x = \pi/7$. Their sum is $6\pi/7$ (B)

40 ~~$1 + \sin x = 2(1 + \cos x)$~~ $\Rightarrow \sin x - 2\cos x = 1$

Graphing $\sin x$ & $2\cos x$ gives:

Also, combining $\sin x - 2\cos x = 1$ w/
 $\sin^2 x + \cos^2 x = 1$ gives
 the solutions $x = \pi/2$ &
 $x = \arccos(-\frac{1}{2}), \arcsin(-\frac{3}{5})$.



$x = \pi/2$ is one solution

~~$2\cos x > 1$~~
 $\sin x - 2\cos x > 1$ for
 $\pi/2 < x < \pi$.

Clearly there is 1 more solution $\pi < x < 3\pi/2$.

2 solutions (B) There are no others