1. We are looking for prime numbers < 40 that leave remainder 1 when divided by 3

$2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37$

$7+13+19+31+37 = \boxed{107}$

2. $\underbrace{(1-2)+(3-4)+\ldots+((2n-1)-2n)}_{n \text{ terms}} = \underbrace{(-1)+(-1)+\ldots+(-1)}_{n \text{ terms}} = n\cdot(-1) = \boxed{-n}$

3. nth triangular number $T_n = \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ $\qquad \frac{35\cdot 36}{2} = \boxed{630}$

4. Each month he adds \$32 to his account. At the end of each month, the amount of interest will be $(1+\frac{.06}{12})(32+A)$ where A is the amount in the bank before his deposit. Writing out the first few terms...

$1.005\cdot 32,\ 1.005(32+1.005\cdot 32),\ 1.005(32+1.005(32+1.005\cdot 32)),\ldots$

$1.005\cdot 32,\ 1.005\cdot 32+1.005^2\cdot 32,\ 1.005\cdot 32+1.005^2\cdot 32+1.005^3\cdot 32,\ldots$

This is the sum of a geometric series with $a_1 = 1.005\cdot 32$, $r = 1.005$, $n = 60$

$\frac{a_1(r^n-1)}{r-1} = \frac{1.005\cdot 32(1.005^{60}-1)}{1.005-1} \approx 2243.804$ to the nearest cent $= \boxed{\$2243.80}$

5. $a_{20}-a_{10} = (a_1+19d)-(a_1+9d) = 10d$

$a_{40}-a_{20} = 20d \rightarrow a_{40}-a_{20} = 2(a_{20}-a_{10}) = 3a_{20}-2a_{10}$

$3\cdot 12-2\cdot 27 = \boxed{-18}$

6. $a_1 = 6,\ a_n = 502$ $\qquad n = \frac{a_n-a_1}{4}+1 = \frac{502-6}{4}+1 = 125$ $\qquad$ sum $= (a_1+a_n)\left(\frac{n}{2}\right) = \frac{508}{2}\cdot 125 =$

$\boxed{31750}$

7. $\lim_{n\to\infty}\left(\frac{\sin(n^2)}{n^2+1}\right)^n = \left[\lim_{n\to\infty}\left(\frac{\sin(n^2)}{n^2+1}\right)\right]^n = \lim_{n\to\infty} 0^n = \boxed{0}$

8. $\sum_{n=0}^{\infty} k^n$ is a geometric series with ratio $= k$

$\boxed{-1 < k < 1}$

9. $r = \frac{1}{\sqrt{2}-1} = \frac{\sqrt{2}+1}{(\sqrt{2}+1)(\sqrt{2}-1)} = \sqrt{2}+1 > 1$ diverges

10. $1^3+2^3+3^3+\ldots+n^3 = \left[\frac{n(n+1)}{2}\right]^2 = (1+2+\ldots+n)^2$ $\boxed{V = 2}$

11. $a_{n+1} = 3a_{n-1} + \frac{1}{2}a_n$ $a_1 = 1, a_2 = 2$ $a_3 = 3\cdot 1 + \frac{1}{2}\cdot 2 = 4$ $a_4 = 3\cdot 2 + \frac{1}{2}\cdot 4 = 8 \ldots$

$a_n = 2^{n-1} \rightarrow a_{1735} = \boxed{2^{1734}}$

12. $1 + \frac{1}{2+\frac{1}{x}} = x \Rightarrow 1 + \frac{x}{2x+1} = x \Rightarrow 2x+1+x = 2x^2+x$

$2x^2 - 2x - 1 = 0 \Rightarrow x = \frac{1+\sqrt{3}}{2}$ (x cannot be negative)

13. $\sum_{m=1}^{10}(m^2+n^2) = \sum_{m=1}^{10} m^2 + \sum_{m=1}^{10} n^2 = \frac{10(10+1)(2\cdot 10+1)}{6} + 10n^2 = 385 + 10n^2$

sum of perfect squares

$\sum_{n=1}^{10}(385+10n^2) = 3850 + 10\cdot 385 = \boxed{7700}$

14. sum of $A = \frac{(p+q)n_A}{2}$ sum of $B = \frac{(p+q)n_B}{2}$

$(p+q)\frac{n_A}{2} - (p+q)\frac{n_B}{2} = p+q$

$\frac{n_A}{2} - \frac{n_B}{2} = 1 \Rightarrow n_A - n_B = 2$

15 $a_n = \frac{a_{n-1}-1}{a_{n-1}+1}$ $a_1 = 2$ $a_2 = \frac{2-1}{2+1} = \frac{1}{3}$ $a_3 = \frac{1/3-1}{1/3+1} = -1/2$ $a_4 = \frac{-1/2-1}{-1/2+1} = -3$

$a_5 = \frac{-3-1}{-3+1} = 2$, etc. a_{1023} is the same as $a_{1023 \bmod 4} = a_3 = \boxed{-1/2}$

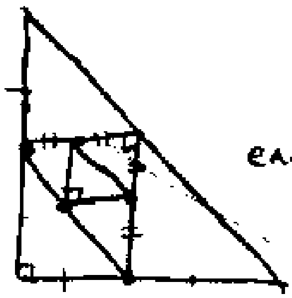
16. $\sum_{k=2}^{\infty}\frac{3}{2^{k+1}} = 3\left(\frac{1}{8}+\frac{1}{16}+\ldots\right) = \frac{3}{4}$

17. Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, $\boxed{233}$

18. $\sin^2 0° + \sin^2 1° + \dots + \sin^2 44° + \sin^2 45° + \sin^2 90° + \sin^2 89° + \dots + \sin^2 46° = \sin^2 0° + \sin^2 1° + \dots + \sin^2 44° + \sin^2 45° + \cos^2 0° + \cos^2 1° + \dots + \sin^2 44° =$

$\underbrace{(1 + 1 + \dots + 1)}_{45 \text{ terms}} + \left(\frac{\sqrt{2}}{2}\right)^2 = 45 + \frac{1}{2} = \boxed{91/}$

19.



each triangle is 1/4 the area of the previous one

Area of 1st triangle $= \frac{12^2}{2} = 72$ $\quad \frac{72}{1 - 1/4} = \boxed{96 \text{ cm}^2}$

20. $\sum_{n=0}^{\infty} \frac{2^n}{n!} = \boxed{e^2}$ $\quad \left(\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x\right)$

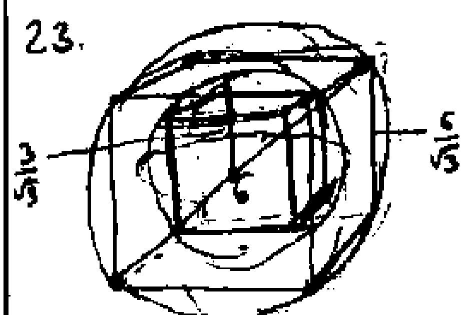
21. as $n \to \infty$, $a_{n-1} \to a_n$ so let $a_{n-1} = a_n = L$

$L = \sqrt{L+8} \to L^2 = L + 8 \to L^2 - L - 8 = 0$

$\boxed{L = \frac{1+\sqrt{33}}{2}}$ (can't be negative)

22. Harmonic series $= \sum_{k=1}^{n} \frac{1}{k}$ $\quad 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \boxed{\frac{25}{12}}$

23.



edge of cube	radius of sphere	#
$6/\sqrt{3}$	3	1
$6/\sqrt{3}^2$	$3/\sqrt{3}$	2
$6/\sqrt{3}^3$	$3/\sqrt{3}^2$	3
$6/\sqrt{3}^4$	$3/\sqrt{3}^3$	4
⋮	⋮	⋮
$6/\sqrt{3}^{21}$	$3/\sqrt{3}^{20}$	21

$r = \frac{3}{\sqrt{3}^{20}} = \frac{3}{3^{10}} = \frac{1}{3^9}$ $\quad S.A. = 4\pi\left(\frac{1}{3^9}\right)^2 = \boxed{\frac{4\pi}{3^{18}}}$

24. $\cot\left(\frac{\pi}{3}\right) = \frac{1}{\tan\left(\frac{\pi}{3}\right)} = \frac{1}{\sqrt{3}}$ so we have an infinite geom. series with $a_1 = 1$ and $r = 1/\sqrt{3}$ . . . $\frac{1}{1-1/\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3}-1} = \boxed{\frac{3+\sqrt{3}}{2}}$

25. $\sum_{k=1}^{\infty} e^{-k} = \frac{e^{-1}}{1-e^{-1}} = \frac{1}{e-1}$ $\ln\left(\frac{1}{e-1}\right) = -\ln(e-1)$ which is $\boxed{\text{finite \& negative}}$

26. $2 \cdot \prod_{k=1}^{\infty} 3^{(5^{-k})} = 2 \cdot 3^{\sum_{k=1}^{\infty} 5^{-k}} = 2 \cdot 3^{\frac{1/5}{1-1/5}} = 2 \cdot 3^{1/4} = \boxed{2\sqrt[4]{3}}$

27. $\frac{1}{n^2-9} = \frac{A}{n-3} + \frac{B}{n+3}$ $1 = A(n+3) + B(n-3)$

$A + B = 0$
$3A - 3B = 1$ $A = 1/6,\ B = -1/6$

$\sum_{n=4}^{\infty} \frac{1}{n^2-9} = \frac{1}{6}\sum_{n=4}^{\infty}\left(\frac{1}{n-3} - \frac{1}{n+3}\right)$

$= \frac{1}{6}\left[\left(1-\cancel{\frac{1}{7}}\right)+\left(\frac{1}{2}-\cancel{\frac{1}{8}}\right)+\left(\frac{1}{3}-\cancel{\frac{1}{9}}\right) + \left(\frac{1}{4}-\cancel{\frac{1}{10}}\right)+\left(\frac{1}{5}-\cancel{\frac{1}{11}}\right)+\left(\frac{1}{6}-\cancel{\frac{1}{12}}\right) + \left(\cancel{\frac{1}{7}}-\cancel{\frac{1}{13}}\right)+\ldots\right]$

$\longleftarrow$ telescoping

$\frac{1}{6}\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}\right)$
$= \frac{1}{6} \cdot \frac{49}{20} = \boxed{\frac{49}{120}}$

28. I is not required, II is sufficient (integral test), III is necessary but not sufficient

$\boxed{\text{II only}}$

29. $\sum_{n=1}^{\infty} \frac{(-1)^n}{3n+1}$ is $\boxed{\text{conditionally convergent}}$ since $\sum\left|\frac{(-1)^n}{3n+1}\right|$ diverges, but $\sum \frac{(-1)^n}{3n+1}$ converges

30. Even if a approaches 0 and $b_n \le a_n$, $\sum b_n$ may not necessarily converge (consider $a_n = \frac{1}{n}$, $b_n = \frac{1}{n+1}$) (which also shows that $\lim_{n\to\infty} b_n \not< \lim_{n\to\infty} a_n$)

b_n does not necessarily diverge either, though it must have a limit ≤ 0

$\sum b_n$ may not necessarily diverge... $\left(a_n = 1/n,\ b_n = \frac{1}{n+1}\right)$ — None of these <u>must</u> be true

31. if $\lim_{n\to\infty} \frac{a_n}{a_{n+1}} = 0$, $a_n < a_{n+1}$ ... a is increasing, so <u>a diverges</u>

32. $\sum_{n=0}^{\infty} \frac{x^{2n}(-1)^n}{(2n)!} = \sin x$ $\sum_{n=0}^{\infty} \frac{2^n(-1)^n}{(2n)!} = \sum_{n=0}^{\infty} \frac{\sqrt{2}^{2n}(-1)^n}{(2n)!} = \boxed{\sin\sqrt{2}}$

33. $.5_7 - .05 + .005_7 - .0005_7 + \dots = \frac{5}{7} - \frac{5}{7^2} + \frac{5}{7^3} - \frac{5}{7^4} + \dots = \frac{5/7}{1+1/7} = 5/8$ in base 10

$5/8$ in base 7 is $\boxed{5_7/11_7}$

34. $\sum_{n=0}^{\infty} \frac{1}{n!} = e \qquad \sum_{n=0}^{\infty} \frac{n}{n!} = \sum_{n=1}^{\infty} \frac{1}{(n-1)!} = e$

$\sum_{n=0}^{\infty} \frac{n^2}{n!} = \sum_{n=1}^{\infty} \frac{n}{(n-1)!} \to \sum_{n=1}^{\infty} \left(\frac{n-1}{(n-1)!} + \frac{1}{(n-1)!}\right) = 2e = \sum_{n=0}^{\infty} \frac{n^2}{n!}$

$= \sum_{n=1}^{\infty} \frac{n-1}{(n-1)!} = e$

$\sum_{n=0}^{\infty} \frac{n^3}{n!} = \sum_{n=1}^{\infty} \frac{n^2}{(n-1)!} = \sum_{n=0}^{\infty} \frac{(n+1)^2}{n!} = \sum_{n=0}^{\infty} \frac{n^2+2n+1}{n!} = \sum_{n=0}^{\infty}\left(\frac{n^2}{n!} + 2\frac{n}{n!} + \frac{1}{n!}\right)$

$= 2e + 2e + e = 5e$

35. $\lim_{b\to\infty} \int_1^b \frac{1}{x^2+1}\,dx = \lim_{b\to\infty} \arctan(b) - \arctan(1) = \pi/4$ (it converges)

thus $\sum_{n=0}^{\infty} \frac{1}{n^2+1}$ converges (the sequence need not start on $n=1$, as long as the terms preceding it are finite)

36. $n^4 - 5n^2 + 4 = (n^2-4)(n^2-1) = (n-1)(n+1)(n-2)(n+2)$

$\frac{A}{n-1} + \frac{B}{n+1} + \frac{C}{n-2} + \frac{D}{n+2} = \frac{1}{n^4-5n^2+4}$

$A(n+1)(n-2)(n+2) + B(n-1)(n-2)(n+2) + C(n-1)(n+1)(n+2) + D(n-1)(n+1)(n-2) = 1$

let $n=1$ .... $-6A = 1$ let $n=-1$ ....: $6B = 1$ let $n=2$ ... $12C = 1$ let $n=-2$ ... $-12D$

$A = -1/6,\ B = 1/6,\ C = 1/12,\ D = -1/12$

$-\sum_{n=3}^{\infty} \frac{1}{6}\left(\frac{1}{n-1} - \frac{1}{n+1}\right) + \sum_{n=3}^{\infty} \frac{1}{12}\left(\frac{1}{n-2} - \frac{1}{n+2}\right)$

$-\frac{1}{6}\left[\left(\frac{1}{2} - \cancel{\frac{1}{4}}\right) + \left(\frac{1}{3} - \cancel{\frac{1}{5}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{6}}\right) + \left(\cancel{\frac{1}{5}} - \cancel{\frac{1}{7}}\right) + \dots\right] + \frac{1}{12}\left[\left(1 - \cancel{\frac{1}{5}}\right) + \left(\frac{1}{2} - \cancel{\frac{1}{6}}\right) + \left(\frac{1}{3} - \cancel{\frac{1}{7}}\right) + \left(\frac{1}{4} - \cancel{\frac{1}{8}}\right) + \left(\cancel{\frac{1}{5}} - \cancel{\frac{1}{9}}\right) + \dots\right]$

$= -\frac{1}{6}\left(\frac{1}{2} + \frac{1}{3}\right) + \frac{1}{12}\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) = \boxed{5/144}$

37. $\ln(x) \approx (x-1) - \frac{(x-1)^2}{2} = -\frac{1}{2}(x^2-4x+3) = -\frac{1}{2}(x-3)(x-1)$

$(\ln(x))^{-1} \approx \frac{-2}{(x-3)(x-1)}$ $\quad \frac{A}{x-3} + \frac{B}{x-1} = \frac{-2}{(x-3)(x-1)}$

$A(x-1) + B(x-3) = -2$

$A=-1, B=1 \implies \int_{4/3}^{5/3} \left(\frac{1}{x-1} - \frac{1}{x-3}\right) dx =$

$\ln|x-1| - \ln|x-3| \Big|_{4/3}^{5/3}$

$= \ln\left|\frac{x-1}{x-3}\right| \Big|_{4/3}^{5/3} = \ln\left|\frac{2/3}{4/3}\right| - \ln\left|\frac{1/3}{5/3}\right|$

$= \boxed{\ln(5/2)}$

38. n^{th} term of Fibonacci $=$

$$\frac{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n}{\sqrt{5}}$$

find F_{16} ... 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, $\boxed{987}$

39. $\frac{S_a(n)}{S_b(n)} = \frac{5n+9}{2n+8}$ $\quad \frac{S_a(1)}{S_b(1)} = \frac{14}{10} = \frac{a_1}{b_1}$ $\quad \frac{S_a(2)}{S_b(2)} = \frac{19}{12} = \frac{a_1+a_2}{b_1+b_2} = \frac{a_1+4}{b_1+b_2}$

$\frac{S_a(3)}{S_b(3)} = \frac{24}{14} = \frac{2a_1+2d_a}{2b_1+2d_b} = \frac{a_2}{b_2} = \frac{4}{b_2}$ $\quad 5a_1 - 7b_1 = 0$ $\quad 19(b_1+b_2) = 12(a_1+4)$

$19(b_1 + 7/3) = 12a_1 + 48$

$\frac{12}{7} = \frac{4}{b_2} \to b_2 = 7/3$ $\quad 12a_1 - 19b_1 = -11/3$

$a_1 = 7/3,\ b_1 = 5/3$

$b_1 = 5/3,\ b_2 = 7/3,\ b_3 = 9/3,\ \boxed{b_4 = 11/3}$

40. $\sum_{x=1}^{\infty} \frac{x^2}{6^x}$ ... $\sum_{x=1}^{\infty} \frac{1}{y^x} = \frac{1}{y-1}$ (take $\frac{d}{dy}$ of both sides) $\to \sum_{x=1}^{\infty} \frac{-x}{y^{x+1}} = \frac{-1}{(y-1)^2}$ (multiply by $-y$)

$\sum_{x=1}^{\infty} \frac{x}{y^x} = \frac{y}{(y-1)^2}$ (take d/dy of both sides) $\to \sum_{x=1}^{\infty} \frac{-x^2}{y^{x+1}} = \frac{(y-1)^2 - 2(y-1)\cdot y}{(y-1)^4} =$

$\frac{-y-1}{(y-1)^3}$ (multiply by $-y$) $\quad \sum_{x=1}^{\infty} \frac{x^2}{y^x} = \frac{y^2+y}{(y-1)^3}$

let $y=6$ $\quad \sum_{x=1}^{\infty} \frac{x^2}{6^x} = \frac{6^2+6}{(6-1)^3} = \boxed{\frac{42}{125}}$