

Mu Alpha Theta National Convention: Denver, 2001
Complex Numbers Topic Test – Mu Division

1. Evaluate: $(5 + 2i\sqrt{3}) - (4 + 3i) + (6 - 2i\sqrt{3}) - (-3 + i\sqrt{3})$
- (A) $-12 + 3i$ (B) $15 + (1 - 2\sqrt{3})i$
(C) $-8 + (2 - \sqrt{3})i$ (D) $10 - (3 + \sqrt{3})i$ (E) NOTA
2. If $x = 4 + i$, $y = 5i - 3$, and $z = 7 + 4i$, what is $\frac{xy - yz}{yx}$?
- (A) $-\frac{15 + 9i}{17}$ (B) $\frac{89 - 31i}{17}$ (C) $\frac{24 - 9i}{34}$ (D) $\frac{13 + 12i}{2}$ (E) NOTA
3. Evaluate: $\sum_{n=1}^{29} (ni^n) =$
- (A) $14 + 15i$ (B) $12 - 8i$ (C) $22 + 28i$ (D) $-12 + 22i$ (E) NOTA
4. Find AB if $\begin{cases} 2A + 5B = 31 - 5i \\ A - B = 5 + i \end{cases}$
- (A) $24 - 8i$ (B) $10 + i$ (C) $13 + 4i$ (D) $-12 + 5i$ (E) NOTA
5. Evaluate: $\begin{vmatrix} 2+i & 3 \\ 1-2i & -4i \end{vmatrix}$
- (A) $5 + i$ (B) $3 - 2i$ (C) $1 - 4i$ (D) $2 + 6i$ (E) NOTA
6. Evaluate: $(2 + 2i)^6$
- (A) 512 (B) $-512i$ (C) -512 (D) $512i$ (E) NOTA
7. Evaluate: $(1 - i)^4(1 - i\sqrt{3})^5$
- (A) $64 - 64i\sqrt{3}$ (B) $64 + 64i\sqrt{3}$ (C) $-64 - 64i\sqrt{3}$ (D) $-64 + 64i\sqrt{3}$ (E) NOTA
8. Which of the following is equal to $4e^{\frac{-5\pi}{3}}$?
- (A) $\text{cis } 60^\circ$ (B) $4 \text{ cis } 60^\circ$ (C) $\text{cis } 120^\circ$ (D) $4 \text{ cis } 120^\circ$ (E) NOTA

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9. Evaluate: $\frac{3e^{\frac{-2\pi}{3i}} \times 8e^{\pi i}}{6e^{\frac{\pi}{3}}}$
- (A) $4e^{\frac{4\pi}{3}}$ (B) $2e^{\pi i}$ (C) $4e^{\frac{\pi}{3}}$ (D) $3e^{\frac{\pi}{3}}$ (E) NOTA
10. What is the norm of $15 + 20i$?
- (A) 20 (B) 25 (C) 30 (D) 35 (E) NOTA
11. Find the sum of an infinite geometric series whose first term is $2 - i$ and common ratio is $\frac{i}{2}$.
- (A) 2 (B) $2 - i$ (C) $2 + i$ (D) $i - 2$ (E) NOTA
12. Which of the following are subsets of the real numbers?
I. the imaginary numbers
II. the integers
III. the complex numbers
IV. the irrational numbers
- (A) II only (B) I & III only
(C) II & IV only (D) I, II, & IV only (E) NOTA
13. For what values of x does $3x^3 + 11x^2 + 7x + 3 = 0$?
- (A) $3, \frac{-1 \pm i\sqrt{2}}{3}$ (B) $-3, \frac{-1 \pm i\sqrt{2}}{3}$ (C) $-3, \frac{-1 \pm \sqrt{2}}{3}$ (D) $3, \frac{\pm 2i\sqrt{2}}{3}$ (E) NOTA
14. Which of the following is a quadratic equation with roots $4 \pm 4i$?
I. $x^2 - 8x + 32 = 0$
II. $x^2 + 16x - 16 = 0$
III. $3x^2 - 24x + 96 = 0$
- (A) I only (B) II only (C) III only (D) I & III only (E) NOTA
15. How many complex roots does $f(x) = 4x^3 + 2x^2 - 3x$ have?
- (A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

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16. What is the sum of the complex roots of $f(x) = 3x^2 - 7x + 12$?
- (A) $-\frac{7}{12}$ (B) $\frac{7}{3}$ (C) -7 (D) 7 (E) NOTA
17. What is the product of the roots of $f(x) = 2x^3 + x^2 + 16x - 9$?
- (A) $\frac{9}{2}$ (B) 16 (C) $\frac{9}{16}$ (D) -8 (E) NOTA
18. What is the sum of the squares of the roots of $x^2 + 8x + 24 = 0$?
- (A) 10 (B) 16 (C) -24 (D) -10 (E) NOTA
19. What is the sum of the reciprocals of the roots of $2x^3 + x^2 - 6x + 7 = 0$?
- (A) $\frac{6}{7}$ (B) $-\frac{6}{7}$ (C) $\frac{7}{6}$ (D) $-\frac{7}{6}$ (E) NOTA
20. A complex number z has the following property: when it is raised to the sixth power it is $(17 - 3i)$ greater than when it is raised to the seventh power. Let A be the number of complex numbers with this property. Let B be the sum of all the complex numbers with this property. Let C be the product of all the complex numbers with this property. What is the value of $A + B + C$?
- (A) $4 - 3i$ (B) $-8 + 3i$ (C) $12 - 3i$ (D) $-9 + 3i$ (E) NOTA
21. Given that $i + 3$ is a root of $x^3 - kx^2 + mx - 5 = 0$ where k and m are real numbers, which of the following are roots?
- I. 1
II. $\frac{1}{2}$
III. $i - 3$
- (A) II only (B) III only (C) I & III only (D) II & III only (E) NOTA
22. The equation $x^3 + kx + y = 0$ has $-2 + 5i$ as one of its roots. What is the value of y ?
- (A) 29 (B) 58 (C) -116 (D) cannot be determined (E) NOTA

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23. What is the polynomial of smallest degree (with a leading coefficient of 1) that has roots $2-i$ and 5 ?

- (A) $x^3 - 9x^2 - 15x - 25 = 0$ (B) $x^3 - x^2 + 12x + 15 = 0$
(C) $x^2 - (7-i)x + 10 - 5i = 0$ (D) $x^2 + (4-3i)x + 9 + 2i = 0$ (E) NOTA

24. Which of the following are possible values for x , if $x^6 = 64$?

- I. $1+i$
II. $i - \sqrt{3}$
III. $1 - i\sqrt{3}$

- (A) I only (B) II only (C) III only (D) II & III only (E) NOTA

25. What is the sum of the square roots of $-i$ added to the sum of the cube roots of i ?

- (A) $-2i$ (B) 0 (C) i (D) $2i$ (E) NOTA

26. What real value(s) of b will ensure that $x^2 - bix - 2 = 0$ has one real root?

- (A) 0 (B) $\pm 2\sqrt{2}$ (C) $\sqrt{2}$ (D) no real value (E) NOTA

27. How many of the following expressions are equivalent to Euler's number?

- I. $\cosh 1 + \sinh 1$
II. $e^{\pi} + 1$
III. $\cos 1 + i \sin 1$
IV. $4 \arctan 1$

- (A) 0 (B) 1 (C) 2 (D) 3 (E) NOTA

28. The hyperbolic sine of the square root of negative one is equal to which of the following?

- (A) $-i \sin 1$ (B) $i \sin 1$ (C) $-\sin i$ (D) $\sin i$ (E) NOTA

29. What is the largest magnitude of a solution to $x^3 - x^2 - 2x - 12 = 0$?

- (A) 3 (B) $3 + 2\sqrt{3}$ (C) $2\sqrt{10}$ (D) $7\sqrt{2}$ (E) NOTA

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30. What is the measure of the smaller angle in the complex plane between the position vectors of the points $4 + 2i$ and $3 - i$?

- (A) $\arctan 2 + \arctan 3$ (B) $\frac{\pi}{4}$
 (C) $\arctan 3 - \arctan 2$ (D) $\frac{\arctan 3}{\arctan 2}$ (E) NOTA

31. A teacher placed an equation of the form $x^2 + bx + c = 0$ on the board to be solved. Joe miscopied the value of c and got $4 \pm i\sqrt{3}$ as the roots. Jim miscopied the value of b , resulting in roots of $\pm 2i$. If Julie copied the problem down correctly, what should she get for roots?

- (A) $4 \pm i\sqrt{2}$ (B) $4 \pm 2\sqrt{3}$ (C) $3 \pm i\sqrt{2}$ (D) $4 \pm 2i\sqrt{5}$ (E) NOTA

32. If the first term of a geometric sequence is 1 and the fifth term is 16, which of the following could be the value of the fourth term?

- I. 4
 II. $4i$
 III. -8
 IV. $8i$

- (A) II only (B) III only (C) I & II only (D) III & IV only (E) NOTA

33. What is the new vector formed when the vector in the complex plane from the origin to $5 - i$ is rotated 45° clockwise about the origin?

- (A) $2\sqrt{2} - 3i\sqrt{2}$ (B) $1 - 5i$ (C) $2\sqrt{3} - i\sqrt{14}$ (D) $3 - i\sqrt{17}$ (E) NOTA

34. Solve for z over the complex numbers if $z^2 - z\sqrt{3} = i$.

- (A) $z \in \left\{ \frac{\sqrt{3} - i}{2}, \frac{2 + i}{2} \right\}$ (B) $z \in \left\{ \frac{\sqrt{3} + i}{2}, \frac{2 + i}{2} \right\}$
 (C) $z \in \left\{ \frac{\sqrt{3} + (1 - \sqrt{3})i}{2}, \frac{2 + i}{2} \right\}$ (D) $z \in \left\{ \frac{(2 + \sqrt{3}) + i}{2}, \frac{(\sqrt{3} - 2) - i}{2} \right\}$ (E) NOTA

35. Solve for z over the complex numbers if $z^4 + 3iz^3 - (4 + i)z^2 - 3iz + 3 + i = 0$.

- (A) $z \in \{1, -1, 1 - 2i, -1 - i\}$ (B) $z \in \{1, -i, 1 + 2i, -1 - i\}$
 (C) $z \in \{1, -1, 1 - i, -1 - 2i\}$ (D) $z \in \{1, -1, 1 + 2i, 1 - i\}$ (E) NOTA

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36. It is given that $1+i$ is a root of the equation $z^2 + (a+2i)z + 5+ib = 0$ where a and b are real. Determine the sum of a and b .

- (A) -4 (B) -3 (C) -2 (D) -1 (E) NOTA

37. For which of the following values of z is $z^i = -1$?

- I. $e^{\frac{\pi}{2}}$
- II. $e^{-\pi}$
- III. $e^{-\frac{\pi}{2}}$

- (A) I only (B) II only (C) III only (D) I & III only (E) NOTA

38. Which of the following are elements of the imaginary numbers?

- I. i
- II. $4-3i$
- III. $\sqrt{-e}$

- (A) I only (B) II only (C) I & II only (D) I & III only (E) NOTA

39. Which of the following is equal to $\ln(-x)$, where x is a positive real number?

- (A) $\ln x + 3\pi i$ (B) $i \ln x$ (C) $i\pi \ln x$ (D) $i(\ln x + \pi)$ (E) NOTA

40. A weight is connected to a spring and damper such that $y'' + 5y' + 7y = 0$, where $y(t)$ is the position of the weight at a given time relative to its equilibrium position. Which of the following is a possible equation for $y(t)$?

- (A) $\frac{1}{2}e^{-\frac{3}{2}t} \cos\left(\frac{\sqrt{3}}{2}t\right)$ (B) $4e^{-\frac{5}{2}t} \sin\left(\frac{\sqrt{3}}{2}t\right)$
- (C) $\frac{3}{2}e^{-\frac{3}{2}t} \cos\left(\frac{5}{2}t\right)$ (D) $7e^{-\frac{5}{2}t} \cos\left(\frac{3}{2}t\right)$ (E) NOTA