

Mu Alpha Theta National Convention: Denver, 2001  
Algebra Topic Test – Euclidean Division

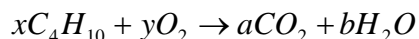
1. If  $f(x) = 3x^4 - 2x^2 + 4x - 15$ , what is  $f(3)$ ?  
(A) 12                      (B) 60                      (C) 222                      (D) 225                      (E) NOTA
  
2. If  $f(x, y) = x^y - y^{xy}$ , what is the value of  $f(2, 3)$ ?  
(A) -721                      (B) -720                      (C) -55                      (D) -50                      (E) NOTA
  
3. What is the justification for the following equality?  
$$b + c = c + b$$
  
(A) distributive law                      (B) associative law of addition  
(C) commutative law of addition                      (D) closure for addition of real numbers (E) NOTA
  
4. How many times will graphs of the equations  $y = x^2 + 3x - 2$  and  $y = 5x^3$  intersect in the Cartesian plane?  
(A) 0                      (B) 1                      (C) 2                      (D) 3                      (E) NOTA
  
5. If  $f(x) = x^2 - 3x$  and  $g(x) = 5x + 2$ , simplify  $f(g(x)) - g(f(x))$ .  
(A) 0                      (B)  $20x^2 + 20x - 4$   
(C)  $30x^2 - 20x$                       (D)  $15x^2 - 15x$                       (E) NOTA
  
6. One long-distance provider charges a \$1.00 connection fee, and \$.04/minute, and another charges a flat \$.05/minute, with no connection fee. A third provider charges \$.03/minute for the first ten minutes, and \$.06/minute for every minute after that, with no connection fee. What is the difference in price between the cheapest and most expensive providers for a twenty-minute call?  
(A) \$0.90                      (B) \$0.80                      (C) \$0.20                      (D) \$0.10                      (E) NOTA
  
7. What is the slope of the line  $\frac{5}{2}x + \frac{3}{2}y = \frac{49}{10}$ ?  
(A)  $5/3$                       (B)  $3/5$                       (C)  $-5/3$                       (D)  $-3/5$                       (E) NOTA
  
8. At what point do the lines  $y = 5x + 1$  and  $y = 3x + 2$  intersect?  
(A) (1,7)                      (B)  $\left(\frac{3}{8}, \frac{113}{40}\right)$                       (C)  $\left(\frac{3}{2}, \frac{13}{2}\right)$                       (D)  $\left(\frac{1}{2}, \frac{7}{2}\right)$                       (E) NOTA

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Algebra Topic Test – Euclidean Division

9. What is the product of the roots of  $2x^2 + 5x - 3 = 0$ ?
- (A) 2                      (B) -2                      (C)  $\frac{3}{2}$                       (D)  $-\frac{3}{2}$                       (E) NOTA
10. What is the distance between the points (-2, 5) and (3, 8)?
- (A)  $\sqrt{10}$                       (B)  $\sqrt{35}$                       (C) 6                      (D)  $\sqrt{34}$                       (E) NOTA
11. The graph of  $f(x) = (x - 5)^2 - 1$  does not pass through which of the quadrants?
- (A) I                      (B) II                      (C) III                      (D) IV                      (E) NOTA
12. What is the slope of the line that is perpendicular to the graph of  $5x + 3y = 13$ ?
- (A)  $\frac{5}{3}$                       (B)  $\frac{3}{5}$                       (C)  $-\frac{3}{5}$                       (D)  $-\frac{5}{3}$                       (E) NOTA
13. Find the roots of  $x^4 - 17x^2 + 16 = 0$ .
- (A) -1, 4                      (B) 1, 4                      (C)  $\pm 1, \pm 4$                       (D) -1, -4                      (E) NOTA
14. Simplify:  $\frac{x^3 - x^2 - 10x - 8}{x^3 + 2x^2 - 11x - 12}$
- (A)  $\frac{x^2 + 2x - 8}{x^2 + x - 12}$                       (B)  $\frac{x^2 - 2x - 8}{x^2 + x - 12}$                       (C)  $\frac{x^2 - 2x + 7}{x^2 + x - 12}$                       (D)  $\frac{x^2 - 2x - 8}{x^2 + 3x - 12}$                       (E) NOTA
15. Billy and Tommy wish to build a paper airplane air force, but they don't know how to fold each other's designs. If it takes two minutes for Billy to fold his design, and ninety seconds for Tommy to fold his design, how long will it take for them to build 500 planes working together?
- (A) 7 hrs, 9 min.                      (B) 7 hrs, 7 min.                      (C) 7 hrs, 6 min.                      (D) 7 hrs, 10 min.                      (E) NOTA

Mu Alpha Theta National Convention: Denver, 2001  
Algebra Topic Test – Euclidean Division

16. Chemists know that  $C_4H_{10}$  (isobutane, a molecule composed of four carbon atoms and ten hydrogen atoms) reacts with  $O_2$  (oxygen, a molecule composed of two oxygen atoms) to form  $CO_2$  (carbon dioxide, a molecule composed of one carbon atom and two oxygen atoms) and  $H_2O$  (water, a molecule composed of two hydrogen atoms and one oxygen atom). In this reaction, each kind of atom remains that kind of atom, for example carbon cannot become hydrogen. Chemists represent this reaction with an equation of the form:



where  $x$ ,  $y$ ,  $a$ , and  $b$  have integral values which cause the number of atoms of each type on each side of the equation to be equal (the number of carbon atoms on the left of the equation is the same as that on the right). In addition, there is no natural number greater than one which is a factor of all of  $x$ ,  $y$ ,  $a$ , and  $b$ . Determine the values of  $x$ ,  $y$ ,  $a$ , and  $b$  and express them as an ordered quadruplet  $(x, y, a, b)$ .

- (A) (2, 13, 8, 10) (B) (2, 15, 8, 10) (C) (1, 6, 3, 5) (D) (1, 7, 4, 5) (E) NOTA
17. The product of two numbers is 378 and their sum is 39. What is the absolute value of the difference between the two numbers?
- (A) 3 (B) 5 (C) 7 (D) 9 (E) NOTA
18. What is the coefficient of  $x^3$  in the expansion of  $(2x-5)^6$ ?
- (A) 20,000 (B) -20,000 (C) 15,633 (D) 1,250 (E) NOTA
19. CDs and DVDs have the same size cases, but a DVD can hold 7 times as much information. A CD holds 70 minutes of music and a DVD holds 490 minutes of music. If David has 186 cases and 31,080 minutes of music, how many DVDs does he have?
- (A) 43 (B) 75 (C) 118 (D) 143 (E) NOTA
20. You have 12 gallons of a 15% sugar-water solution. How much more water must you add to get it to be a 7.4% sugar-water solution, rounded to the nearest tenth of a gallon?
- (A) 12.3 (B) 14.5 (C) 18.7 (D) 24.3 (E) NOTA
21. A teacher has a system of awarding cards to his students to reward good scores on tests. For each score between 80 and 90, the student receives a white card. For each score between 91 and 100, the student receives a yellow card. Two white cards can be traded in for a yellow card. Five yellow cards can be traded in for a blue card. Three blue cards can be traded in for a five pieces of candy. What is the price of each piece of candy in white cards?
- (A) 4 (B) 5 (C) 6 (D) 7 (E) NOTA

Mu Alpha Theta National Convention: Denver, 2001  
Algebra Topic Test – Euclidean Division

22. What is the quotient when  $x^5 - 5x^4 + 11x^3 - 23x^2 + 25x - 3$  is divided by  $x - 3$ ?

- (A)  $x^4 + 2x^3 + 5x^2 - 8x + 1$       (B)  $x^4 - 2x^3 + 5x^2 - 8x + 1$   
 (C)  $x^4 + 2x^3 + 5x^2 - 8x - 1$       (D)  $x^4 + 2x^3 - 5x^2 - 8x + 1$       (E) NOTA

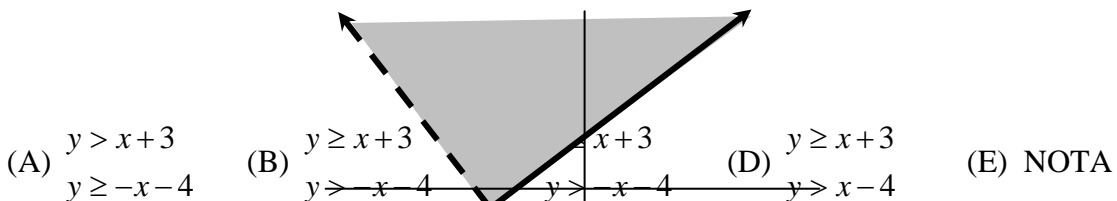
23. In a given function,  $y$  is directly proportional to  $k$  and inversely proportional to  $h$ . Which of the following is an equation that satisfies these conditions?

- (A)  $y = \frac{x}{hk}$       (B)  $y = \frac{hx}{k}$       (C)  $y = h k x$       (D)  $y = \frac{kx}{h}$       (E) NOTA

24. What is the value of the discriminant of  $y = 2x^2 - 3x + 5$ ?

- (A) -31      (B) -11      (C) 14      (D) 39      (E) NOTA

25. What set of equations could describe the graph below?



26. A hotel currently has a swimming pool that takes three hours to drain and four hours to fill. However, they are in the process of building a pool that is capable of holding twice the current volume of water. To accommodate the new filtration/purification system required to bring the system up to code, the radius of the drainage pipe had to be doubled. But to cut down on costs, they keep the current filling hoses. How long will it take to drain and refill the new pool (a task that must be performed once each month to satisfy the Department of Health)? Assume that the drain time is directly proportional to the cross-sectional area of the drainpipe.

- (A) 14 hours      (B) 8.75 hours      (C) 11 hours      (D) 9.5 hours      (E) NOTA

Mu Alpha Theta National Convention: Denver, 2001  
Algebra Topic Test – Euclidean Division

27. Mike and Ike decide to do a little racing, but Ike is slower, so he's given a head start. Ike passes the one-mile mark doing sixty miles an hour. Eleven minutes later, Mike passes the same point traveling 71 miles per hour. How many minutes after the start of the race will Mike catch up to Ike, assuming Mike and Ike travel at constant speeds?

- (A) 57                      (B) 60                      (C) 63                      (D) 51                      (E) NOTA

28. Simplify:  $\frac{\frac{1}{ab} + \frac{2}{bc} + \frac{3}{ac}}{\frac{2a+3b+c}{abc}}$

- (A) 1                      (B)  $\frac{a+b+c}{abc}$                       (C)  $\frac{abc}{a+b}$                       (D)  $\frac{1}{abc}$                       (E) NOTA

29. Which of the following equations will give the following data points

<b>X</b>	<b>1</b>	<b>3</b>	<b>5</b>	<b>7</b>
<b>Y</b>	<b>3</b>	<b>11</b>	<b>27</b>	<b>51</b>

- (A)  $y = 2x + 1$                       (B)  $y = 8x - 5$                       (C)  $y = x^2 + 2$                       (D)  $y = 4x^2 - 3x - 3$                       (E) NOTA

30. A baseball player currently has 50 hits in 317 at-bats. What is the minimum number of hits he needs in order to achieve an overall average of one hit in every four at-bats?

- (A) 37                      (B) 39                      (C) 41                      (D) 43                      (E) NOTA

31. Which of these terms is not a method by which estimates of unknown values of an unknown function can be determined using known values of the function?

- (A) interpolation                      (B) DeMorgan's Law  
(C) extrapolation                      (D) linear regression                      (E) NOTA

32. What are the roots of  $x^2 - x - 6 = 36$ ?

- (A) 7, -6                      (B) -7, 6                      (C) -2, 3                      (D) 2, -3                      (E) NOTA

33. A semi-truck driver's eyes are about 9 feet above the ground, but the hood of his truck extends 8 feet in front of him and 6 feet above the ground, thus obscuring part of his view. What is the minimum distance at which a dime can be on the ground in front of the driver and still be visible to him?

- (A) 12 feet                      (B) 18 feet                      (C) 24 feet                      (D) 36 feet                      (E) NOTA

**Mu Alpha Theta National Convention: Denver, 2001**  
**Algebra Topic Test – Euclidean Division**

34. Tom, Bill, and Brian order two large pizzas. Tom eats two slices, while Bill and Brian both have three. If two-thirds of the total amount of pizza is remaining and assuming each of the slices represents an equal portion of the pizza, how many slices were in originally in each pizza?

- (A) 30                      (B) 24                      (C) 18                      (D) 12                      (E) NOTA

35. A man is presently nine times as old as his son is. In 9 years, he will be three times as old as his son. How old is the son now?

- (A) 3                      (B) 4                      (C) 5                      (D) 6                      (E) NOTA

36. What is the sum of the abscissas of the following set of points?

<b>x</b>	<b>1.2</b>	<b>2.7</b>	<b>3.0</b>	<b>5.8</b>	<b>6.7</b>	<b>8.0</b>	<b>12.7</b>	<b>15.9</b>	<b>17.2</b>
<b>y</b>	<b>10.0</b>	<b>10.0</b>	<b>9.8</b>	<b>8.2</b>	<b>7.6</b>	<b>7.4</b>	<b>5.1</b>	<b>4.3</b>	<b>2.8</b>

- (A) 65.2                      (B) 67.4                      (C) 70.3                      (D) 73.2                      (E) NOTA

37. The activity,  $A$ , of a radioactive nuclide as a function of time is given by the equation

$$A = A_0 e^{-\lambda t}, \text{ where } \lambda = \frac{\ln 2}{t_{1/2}} \text{ and } A_0 \text{ is the activity at } t = 0. \text{ } t_{1/2} \text{ is called the } \textit{half-life} \text{ of the}$$

nuclide, and is the length of time it takes for one-half of a sample to become non-radioactive. If  $^{239}\text{U}$  has an initial activity of 24,000 and a half-life of 23.5 minutes, what will its activity be after 20 minutes rounded to the nearest one?

- (A) 15,327                      (B) 14,026                      (C) 13,305                      (D) 12,345                      (E) NOTA

38. Mr. Toad invested \$250,000 in January of 1999. The progress of his investments is listed in the table below, with each percentage up or down in reference to the point preceding it.

<u>Date</u>	<u>Up / Down</u>	<u>%</u>
March '99	Up	25
June '99	Down	15
September '99	Down	13
December '99	Up	5
March '00	Down	6
June '00	Up	35
September '00	Up	12
December '00	Down	3

