ALPHA TRIGONOMETRY TOPIC TEST 2001 MAO CONVENTION

(1) THE INESTIMABLE BUDDHA ONCE NOTED THAT THERE ARE 3 IMMUTABLE, INCONTROVERTIBLE TRUTHS: "A PINT'S A POUND THE WORLD AROUND", "GEORGE W. BUSH WILL NEVER SENSIBLY COMPLETE THREE SENTENCES IN A ROW" AND "$\sin^2 x + \cos^2 x = 1$ FOR ALL x". GIVEN HIS EXEMPLARY ACCURACY ON THE FIRST TWO COUNTS, WE'LL GIVE HIM THE BENEFIT OF THE DOUBT ON THE 3^{RD}. (A)

(2) $78^\circ \cdot \frac{\pi}{180^\circ} = \frac{13\pi}{30}$ (D)

(3) $0 - \frac{\sqrt{3}}{2} + 1 - 2 + 1 + \frac{\sqrt{3}}{3} = -\frac{\sqrt{3}}{6}$ (D)

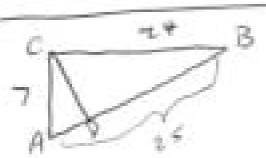
(4) $1 + \cot^2 42^\circ = \csc^2 42 = \sec^2 48^\circ$ (C)

(5) (A)

(6) $\frac{57\pi}{36} \cdot \frac{180}{\pi} = 285^\circ$ (B)

(7) $\arctan\left(\sin \frac{\pi}{2}\right) = \arctan(1) = \pi/4$ (A)

(8) $90^\circ - 25^\circ 43' 37'' = 64^\circ 16' 23''$ (B)

(9) 

$\sin\angle BCD + \csc\angle ACD = \frac{BD}{BC} + \frac{AC}{AD} = \frac{BC}{AB} + \frac{AB}{AC} = \frac{BC\cdot AC + AB^2}{AC\cdot AB}$

$= \frac{168 + 625}{7\cdot 25} = \frac{793}{175}$ (B)

(10) $C = 8\pi$; $\frac{\theta}{2\pi} = \frac{0.5}{8\pi}$; $\theta = \frac{1}{8}$ (C)

(11) $5x - 6 = 9(\sec^2\theta - \tan^2\theta) = 9 \therefore x = 3$ (C)

(12) By Law of Sines: $\frac{AB}{\sin ACB} = \frac{BC}{\sin BAC} \therefore BC = \frac{\sin 65^\circ}{\sin 25^\circ}$ (C)

(13) Periods of $\cos\frac{x}{4}$ & $\sin\frac{x}{2}$ are 8π, 4π resp. $\therefore$ The period of their sum is 8π (B)

(14) $\sin\left(\theta - \frac{3\pi}{2}\right) + \cos(\pi + \theta) = \sin\theta\left(\cos\frac{3\pi}{2}\right) - \sin\frac{3\pi}{2}\cos\theta + \cos\pi\cos\theta - \sin\pi\sin\theta = 0$ (A)

(15) $\frac{\tan x \csc x - \cos x}{1 - \cos^2 x} = \frac{\frac{1}{\cos x} - \cos x}{1 - \cos^2 x} = \frac{\frac{1 - \cos^2 x}{\cos x}}{1 - \cos^2 x} = \sec x$ (C)

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(16) $z^{14} = \left[2\left(e^{i\pi/7}\right)\right]^{14} = 2^{14}e^{2\pi i} = 2^{14}$ (D)

(17) Expression $= \ln|\sec^2 u - \tan^2 u| + e^{\ln 1} = \ln 1 + e^0 = 1$ (C)

(18) $3600 \cdot \frac{\pi}{2} = 1800\pi$ ∴ The graph of $A(t)$ goes through 900 periods in the hour ∴ Brett takes 900 breaths in the hour. (E)

(19) $\cos(-u) = \cos(u)$; $\sin(-u) = -\sin(u)$ ∴ $\cos(-u) + \sin(-u)\cos(-u) + \sin^2(-u) =$
$\cos u - \sin u \cos u + \sin^2 u = \frac{1}{\sqrt{3}} - \frac{2}{3}\left(\frac{1}{\sqrt{3}}\right) + \left(\frac{\sqrt{2}}{\sqrt{3}}\right)^2 = \frac{6+\sqrt{3}}{9}$ (D)

(20) $\cos 2925° - \tan 2400° + \csc 2970° = \cos 45° - \tan 240° + \csc 90° =$
$\frac{\sqrt{2}}{2} - \sqrt{3} + 1 =$ (A)

(21) $\frac{43\pi}{3} = 14\pi + \frac{\pi}{3}$ $\quad \frac{31\pi}{2} = 15\pi + \frac{\pi}{2} \rightarrow 14\pi + \frac{\pi}{3} < x < 15\pi + \pi/2$, (D)
$\sin x$ increases from $14\pi + \pi/3$ to $14\pi + \pi/2$, then decreases the rest of the way

(22) $4\cos 4t - 4\sqrt{3}\sin 4t = 8\left[\frac{1}{2}\cos 4t - \frac{\sqrt{3}}{2}\sin 4t\right] = 8\left[\cos 60°\cos 4t - \sin 60°\sin 4t\right] =$
$8\cos(4t + \pi/3) = 8\cos(4t - 5\pi/3)$ (C)

(23) $= \frac{27}{27 \cdot 18} \cdot \frac{e^{i(41°)}e^{i(64°)}}{e^{i(27°)}e^{i(18°)}} = \frac{1}{18}\left(e^{i(41° + 64° - 27° - 18°)}\right) = \frac{1}{18}e^{i60°} =$ ᴬ
$\frac{1}{18}\left(\cos 60° + i\sin 60°\right) = \frac{1}{18}\left[\frac{1}{2} + \frac{i\sqrt{3}}{2}\right]$ (B)

(24) $\frac{\tan x}{1+\sec x} + \frac{1+\sec x}{\tan x} = \frac{\tan^2 x + (1+\sec x)^2}{(\tan x)(1+\sec x)} = \frac{(\tan^2 x + 1) + \sec^2 x + 2\sec x}{(\tan x)(1+\sec x)} =$
$\frac{2\sec^2 x + 2\sec x}{(\tan x)(1+\sec x)} = \frac{2\sec x(1+\sec x)}{\tan x(1+\sec x)} = 2\csc x.$ (A)

ᴬ ~~YOU'RE FRAUGHT W/ THE~~ DANGER OF A FREUDIAN SLIP.

(25) I: $\cos 2x + \sin^2 x = \cos^2 x \geq 0$ for all x.
II: $\sec^2 x + 2\tan x = \frac{1 + 2\sin x\cos x}{\cos^2 x} = \frac{1+\sin 2x}{\cos^2 x} \geq 0$ for all x (since $\sin 2x \geq -1$)
III: For $x = \arctan(-5)$, $\tan^3 x + 5 = -120$.
IV: $4\cos^2 x - \sin^2 2x = 4\cos^2 x - 4\sin^2 x\cos^2 x = 4(\cos^2 x)(1-\sin^2 x) \geq 0$ for All x. (C)
I, II, IV

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(26) $\cos^3 x \sin^4 x = \cos x\,(1-\sin^2 x)\sin^4 x = \cos x\,(\sin^4 x - \sin^6 x) = \cos x\,(u^4 - u^6)$ C

(27) $\sin^2 \frac{\pi}{12} + \cos\frac{\pi}{6} + \tan\frac{\pi}{4} = \frac{1-\cos\frac{\pi}{6}}{2} + \frac{\sqrt{3}}{2} + 1 = \frac{3}{2} + \frac{\sqrt{3}}{4} = \frac{6+\sqrt{3}}{4}$ D

(28) $\sin y = \frac{\sqrt{2}}{2} \Rightarrow y = 45^\circ$ $\quad \frac{\tan x + \tan y}{1-\tan x \tan y} = \sqrt{3} = \tan(x+y) \Rightarrow x+y = 60^\circ \Rightarrow x = 15^\circ$ B

(29) $\sin^2 5\varphi = 1 \therefore 5\varphi = \frac{\pi}{2} + 2n\pi$ & $5\varphi = \frac{3\pi}{2} + 2m\pi$, or

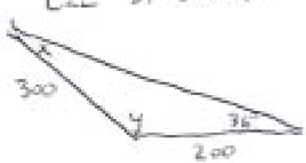
$\varphi = \frac{\pi}{10} + \frac{2n\pi}{5}$ & $\varphi = \frac{3\pi}{10} + \frac{2m\pi}{10}$.

Under the restriction $\frac{7\pi}{5} < \varphi < \frac{9\pi}{5}$, we find sol'ns $m = 1,2,3$, & $n = 2,3,4$.
The sum of these solutions is $3\left(\frac{\pi}{10} + \frac{3\pi}{10}\right) + \frac{2\pi}{10}(1+2+3+2+3+4) = \frac{36\pi}{5}$ B

(30) From Law of Sines: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R \therefore a = 4\sin 45^\circ$, $b = 4\sin 30^\circ$
$c = 4\sin 105^\circ = 4\sqrt{\frac{1-\cos 210^\circ}{2}} = 4\sqrt{\frac{1+\sqrt{3}/2}{2}} = 2\sqrt{2+\sqrt{3}} = 2\left(\frac{\sqrt{6}+\sqrt{2}}{2}\right) = \sqrt{6}+\sqrt{2}$.
$a = 2\sqrt{2}$, $b = 2 \therefore$ $P = 2 + 3\sqrt{2} + \sqrt{6}$ B

(31) Let $x = \sin 2\theta = 2\sin\theta\cos\theta \therefore$ given eqn is $4x^2 - 6x + 11 = 7 + 5x - 2x^2 \therefore$
$6x^2 - 11x + 4 = (2x-1)(3x-4) = 0 \therefore x = 1/2$ or $x = 4/3$. Latter produces no sol'ns for θ. Former gives $\theta = \pi/12, 5\pi/12, 13\pi/12, 17\pi/12$ in $[\pi/12, 25\pi/12)$ D

(32) Law of Sines: $\frac{\sin x}{200} = \frac{\sin 36^\circ}{300}$



(33) $y = \cos x - \sin 2x - \cos 3x = \cos x - 2\sin x\cos x - [\cos 2x\cos x - \sin 2x\sin x] =$
$\cos x - 2\sin x\cos x - (1-2\sin^2 x)\cos x + 2\sin^2 x\cos x = \cos x[4\sin^2 x - 2\sin x] =$
$2\cos x\sin x[2\sin x - 1] \therefore y = 0$ when $\cos x = 0$, $\sin x = 0$, $\sin x = 1/2$.
The first has 4 sol'ns on the interval, the 2^{nd} has 5, and the last 4.
13 total. B

(34) Law of sines: $\frac{8}{\sin\pi/6} = 2R \therefore R = 8$, Area $= 64\pi$ D

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35 $\tan A = \frac{28}{45}$, $\tan B = -\frac{20}{21}$ $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{28}{45} - \frac{20}{21}}{1 + \frac{28}{45}\cdot\frac{20}{21}} = -\frac{312}{1505}$ B

36 There are $\binom{6}{2} = 15$ ways to choose a pair; only one satisfies the given criterion $\frac{1}{15} \to$ A

37 $(-r, \theta + 5\pi) = (-r, \theta + \pi) = (r, \theta + \pi - \pi) = (r, \theta)$ A

38 $K = \frac{1}{2}ab\sin\theta$, so the sought ratio is $\frac{\sin 2\theta}{\sin\theta} = \frac{2\cos\theta\sin\theta}{\sin\theta} = 2\cos\theta =$
$= 2\left(\frac{55}{73}\right) = \frac{110}{73}$ C

39 Period is $\frac{2\pi}{(3)} = \frac{2\pi}{3}$ D

40 $K = \frac{1}{2}ab\sin\theta = \frac{1}{2}(\sqrt{6})(2\sqrt{3})\sin\theta = 3\sqrt{2}\sin\theta$.
Max value of $\sin\theta$ is 1, so max area is $3\sqrt{2}$ B