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Logarithms and Exponents Topic Test Solutions – Alpha Division

1. Given that $\frac{\log_{10} A}{\log_{10} B} = \frac{A}{B} = \frac{2}{3}$, what are *A* and *B*, in that order?

   (A) 2 and 3 (B) 1 and $\frac{3}{2}$ (C) $\left(\frac{2}{3}\right)^4$ and $\left(\frac{2}{3}\right)^3$ (D) $\left(\frac{2}{3}\right)^3$ and $\left(\frac{2}{3}\right)^2$ (E) NOTA

   $\frac{\log_{10} A}{\log_{10} B} = \frac{2}{3} \Rightarrow \log_B A = \frac{2}{3} \Rightarrow B^{\frac{2}{3}} = A, and\ \frac{A}{B} = \frac{2}{3} \Rightarrow A = \frac{2B}{3}$, so we get

   $B^{\frac{2}{3}} = \frac{2B}{3} \Rightarrow B^{\frac{1}{3}} = \frac{3}{2} \Rightarrow B = \left(\frac{3}{2}\right)^3$, and $A = \frac{2B}{3} = \frac{2}{3}\left(\frac{3}{2}\right)^3 = \left(\frac{3}{2}\right)^2$, so we get E.

2. Solve for *x*: $\log_5(\log_3(\log_6 x)) = 0$

   (A) 216 (B) 125 (C) 36 (D) 243 (E) NOTA

   $\log_5(\log_3(\log_6 x)) = 0 \Rightarrow \log_3(\log_6 x) = 1 \Rightarrow \log_6 x = 3 \Rightarrow x = 6^3 = 216$, so we get A.

3. Evaluate: $\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\ldots}}}}}$

   (A) 3 and -2 (B) 3 (C) -2 (D) 6 (E) NOTA

   Consider

   $x = \sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\ldots}}}}} \Rightarrow x^2 = 6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\ldots}}}} = 6 + x \Rightarrow$

   $x^2 - x - 6 = 0 \Rightarrow (x-3)(x+2) = 0 \Rightarrow x = 3 or x = -2,$

   but we can count out the negative case, since the sum is positive by inspection. So we get B.

4. Which of the following is equal to i^i?

   (A) $e^{i\pi}$ (B) e^{π} (C) $e^{-\frac{\pi}{2}}$ (D) $e^{-i\pi}$ (E) NOTA

   Using $e^{\frac{i\pi}{2}} = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = i$, we get $i^i = \left(e^{\frac{i\pi}{2}}\right)^i = e^{\frac{i^2\pi}{2}} = e^{\frac{-\pi}{2}}$, which is C.

5. Solve for x: $a^x b^x = c$

(A) $\frac{\ln c}{\ln a - \ln b}$ (B) $\frac{\ln c}{\ln(a+b)}$ (C) $\log_{ab} c$ (D) $\frac{\ln c}{\ln a \ln b}$ (E) NOTA

$a^x b^x = c \Rightarrow (ab)^x = c \Rightarrow \ln(ab)^x = \ln c \Rightarrow x\ln(ab) = \ln c \Rightarrow x = \frac{\ln c}{\ln(ab)} = \log_{ab} c$, since $\frac{\log_m x}{\log_m y} = \log_y x$, so we get C.

6. Solve for x: $\frac{2}{3}\ln x^3 + \frac{1}{2}\ln x^4 - \frac{1}{2}\ln x^2 = 6$

(A) e (B) e^3 (C) 3^6 (D) e^2 (E) NOTA

$3\ln x = 2\ln x + 2\ln x - \ln x = \frac{2}{3} \bullet 3\ln x + \frac{1}{2} \bullet 4\ln x - \frac{1}{2} \bullet 2\ln x = \frac{2}{3}\ln x^3 + \frac{1}{2}\ln x^4 - \frac{1}{2}\ln x^2 = 6 \Rightarrow$
$\ln x = 2 \Rightarrow x = e^2$
This gives us D.

7. Solve for x: $18^{x^2+2x+4} = \left(54\sqrt{2}\right)^{x^2+4}$

(A) 2 (B) 4 (C) 3 (D) $\frac{3}{2}$ (E) NOTA

$\left(3\sqrt{2}\right)^{2x^2+4x+8} = \left(\left(3\sqrt{2}\right)^2\right)^{x^2+2x+4} = 18^{x^2+2x+4} = \left(54\sqrt{2}\right)^{x^2+4} = \left(\left(3\sqrt{2}\right)^3\right)^{x^2+4} = \left(3\sqrt{2}\right)^{3x^2+12} \Rightarrow$
$2x^2 + 4x + 8 = 3x^2 + 12 \Rightarrow x^2 - 4x + 4 = 0 \Rightarrow (x-2)^2 = 0 \Rightarrow x = 2$
Which gives us A.

8. What is the product of the two solutions for x: $-\frac{2}{3}\log_x a + \frac{4}{3}\log_{ax} a + \frac{7}{3}\log_{a^2x} a = 0$

(A) a (B) $\sqrt[3]{a^2}$ (C) a^{-1} (D) $a^{-\frac{4}{3}}$ (E) NOTA

$\log_c d = \frac{\ln d}{\ln c} = \frac{1}{\frac{\ln c}{\ln d}} = \frac{1}{\log_d c}$, so

$$-\frac{2}{3\log_a x} + \frac{4}{3\log_a ax} + \frac{7}{3\log_a a^2x} = -\frac{2}{3}\log_x a + \frac{4}{3}\log_{ax} a + \frac{7}{3}\log_{a^2x} a = 0 =$$

$$-\frac{2\log_a ax \log_a a^2x}{3\log_a x \log_a ax \log_a a^2x} + \frac{4\log_a x \log_a a^2x}{3\log_a x \log_a ax \log_a a^2x} + \frac{7\log_a x \log_a ax}{3\log_a x \log_a ax \log_a a^2x} \Rightarrow$$

$$-2(1+\log_a x)(2+\log_a x) + 4(\log_a x)(2+\log_a x) + 7(\log_a x)(1+\log_a x) =$$

$$-2(\log_a a + \log_a x)(\log_a a^2 + \log_a x) + 4(\log_a x)(\log_a a^2 + \log_a x) + 7(\log_a x)(\log_a a + \log_a x) =$$

$$-2\log_a ax \log_a a^2x + 4\log_a x \log_a a^2x + 7\log_a x \log_a ax = 0 \Rightarrow$$

$$-4 - 6\log_a x - 2(\log_a x)^2 + 8\log_a x + 4(\log_a x)^2 + 7\log_a x + 7(\log_a x)^2 = 0 \Rightarrow$$

$$-4 + 9\log_a x + 9(\log_a x)^2 = 0 \Rightarrow \log_a x = \frac{-9 \pm \sqrt{81-4(9)(-4)}}{18} = \frac{-9 \pm \sqrt{81+144}}{18} = \frac{-9 \pm 15}{18} \Rightarrow$$

$$x = a^{\frac{-9\pm15}{18}}, and, a^{\frac{-9-15}{18}} \bullet a^{\frac{-9+15}{18}} = a^{\frac{-18}{18}} = a^{-1}$$

This gives us C.

9. If \$52,000 is invested at a 4% annual rate compounded continuously, how many years (to the nearest year) will it take for it to triple?

(A) 31 (B) 32 (C) 19 (D) 27 (E) NOTA

$52000e^{.04t} = m(t)$, where m(t) is the money at time t(measured in years), so we must solve the following equation: $52000e^{.04t} = 3 \bullet 52000 \Rightarrow e^{.04t} = 3 \Rightarrow .04t = \ln 3 \Rightarrow t = \frac{\ln 3}{.04} \approx 27 years$, which is D.

10. Evaluate: $\frac{1}{\log_{\left(\frac{1}{3}\right)}144}+\frac{1}{\log_3 144}+\frac{1}{\log_{12}144}$

(A) $\frac{2}{3}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 1 (E) NOTA

Using the formula that we derived for 8, we get

$$\frac{1}{\log_{\left(\frac{1}{3}\right)}144}+\frac{1}{\log_3 144}+\frac{1}{\log_{12}144}=\log_{144}\frac{1}{3}+\log_{144}3+\log_{144}12=-\log_{144}3+\log_{144}3+\log_{144}12=\frac{1}{2}$$

Which gives us B

11. Which of the following is equivalent to: $\frac{1}{2}\ln(9)+\ln(2)+\frac{1}{3}\ln(8^2)$

(A) $\ln\left(\frac{167}{6}\right)$ (B) $\ln(167)+\ln(6)$ (C) $\frac{11}{6}\ln(1152)$ (D) $\ln(3)+\ln(8)$ (E) NOTA

$\frac{1}{2}\ln(9)+\ln(2)+\frac{1}{3}\ln(8^2)=\ln 9^{\frac{1}{2}}+\ln 8^{\frac{1}{3}}+\frac{2}{3}\ln 8=\ln 3+\frac{1}{3}\ln 8+\frac{2}{3}\ln 8=\ln 3+\ln 8$, which is D.

12. How many solutions are there to the following problem: $x * x^{\frac{1}{x}}=x^x$

(A) 0 (B) 1 (C) 2 (D) 3 (E) NOTA

By inspection, x=1 is a solution. If $x^{1+\frac{1}{x}}=x * x^{\frac{1}{x}}=x^x$, then since
$x\neq 1$ $\log_x x^{1+\frac{1}{x}}=\log_x x^x=1+\frac{1}{x}=x\Rightarrow x^2-x-1=0$,
which has solutions of $\frac{1+\sqrt{5}}{2}$ and $\frac{1-\sqrt{5}}{2}$. Thus we get 3 solutions, which is D.

13. If you multiply together the solutions from problem 12, what do you get?

(A) -2 (B) -1 (C) $\frac{1+\sqrt{5}}{2}$ (D) $\frac{1-\sqrt{5}}{2}$ (E) NOTA

Multiplying the above answers gives you $1\bullet\frac{1+\sqrt{5}}{2}\bullet\frac{1-\sqrt{5}}{2}=\frac{1-5}{4}=-1$, which is B.

14. $A\left(2^{\frac{x}{y}}\right) = B$. Solve for x with respect to y.

(A) $y\log_2 A$ (B) $y\log_2\left(\frac{B}{A}\right)$ (C) $Ay^2\log_2 B$ (D) $\log_2\left(\frac{Ay}{B}\right)$ (E) NOTA

$A\left(2^{\frac{x}{y}}\right) = B \Rightarrow 2^{\frac{x}{y}} = \frac{B}{A} \Rightarrow \frac{x}{y} = \log_2 2^{\frac{x}{y}} = \log_2\left(\frac{B}{A}\right) \Rightarrow x = y\log_2\left(\frac{B}{A}\right)$, which is B.

15. Determine the sum of all solutions to $3^{x\log_3(x+2)} = x+2$.

(A) 2 (B) 1 (C) 0 (D) 3 (E) NOTA

$3^{x\log_3(x+2)} = x+2 \Rightarrow x\log_3(x+2) = \log_3 3^{x\log_3(x+2)} = \log_3(x+2) \Rightarrow (x-1)\log_3(x+2) = 0 \Rightarrow$
$x = 1 or \log_3(x+2) = 0 \Rightarrow x+2 = 1 \Rightarrow x = -1$
And thus the solutions sum to 0, which is C.

16. Given that $0 \le x < 2\pi$, solve for x: $3^{\sin x} = \frac{1}{3}$

(A) $\frac{3\pi}{2}$ (B) $\frac{\pi}{2}$ (C) $-\pi$ (D) $\frac{-\pi}{3}$ (E) NOTA

$3^{\sin x} = \frac{1}{3} \Rightarrow \sin x = \log_3 3^{\sin x} = \log_3 \frac{1}{3} = -1$, and with $0 \le x < 2\pi$, $\sin x = -1 \Leftrightarrow x = \frac{3\pi}{2}$, which is A.

17. Given that $0 \le x < 2\pi$, determine the sum of all values of x for which $\ln(\sin x) - \ln(\cos x) = 0$.

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{3\pi}{2}$ (E) NOTA

$\ln(\tan x) = \ln\left(\frac{\sin x}{\cos x}\right) = \ln(\sin x) - \ln(\cos x) = 0 \Rightarrow \tan x = 1$, so with $0 \le x < 2\pi$, we get $x = \frac{\pi}{4}, \frac{5\pi}{4}$, however we can rule out $x = \frac{5\pi}{4}$, since $\sin\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2} = \cos\left(\frac{5\pi}{4}\right)$ and we can't

take the natural log of a negative number. Therefore, we are left with $x = \frac{\pi}{4}$, so we get A.

18. What is the product of the solutions for x? $\left(2\log_4 x^{\log_4 x}\right) - \left(4\log_4 x\right) + 3 = 9$

(A) 64 (B) 32 (C) -6 (D) 16 (E) NOTA

$$2\log_4 x^{\log_4 x - 2} + 3 = 2\log_4 \frac{x^{\log_4 x}}{x^2} + 3 = 2\log_4 x^{\log_4 x} - 2\log_4 x^2 + 3 = \left(2\log_4 x^{\log_4 x}\right) - \left(4\log_4 x\right) + 3 = 9$$
$$\Rightarrow 2\log_4 x^{\log_4 x - 2} = 6 \Rightarrow (\log_4 x)^2 - 2\log_4 x = (\log_4 x - 2)\log_4 x = \log_4 x^{\log_4 x - 2} = 3 \Rightarrow$$
$$(\log_4 x)^2 - 2\log_4 x - 3 = 0 \Rightarrow \log_4 x = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2 \pm 4}{2} = 3, -1 \Rightarrow x = 4^3, 4^{-1}$$

Multiplying these together, we get $4^3 \bullet 4^{-1} = 4^2 = 16$, which is D.

19. Which of the following is equal to $9^9 + 9^9 + 9^9 + 9^9 + 9^9 + 9^9 + 9^9 + 9^9 + 9^9$?

(A) 3^{22} (B) 3^{19} (C) 3^{24} (D) 3^{20} (E) NOTA

$9^9 + 9^9 + 9^9 + 9^9 + 9^9 + 9^9 + 9^9 + 9^9 + 9^9 = 9\left(9^9\right) = 9^{10} = \left(3^2\right)^{10} = 3^{20}$, which is D.

20. Solve for x: $4^x - 16(2^x)^2 + 64 \cdot 4^x = 98$

(A) 2 (B) 1 (C) $\frac{1}{2}$ (D) $\frac{3}{4}$ (E) NOTA

$$49 \cdot 4^x = (65-16)4^x = 65(4^x) - 16(2^2)^x = 63 \bullet 4^x - 16(2^{2x}) = 4^x - 16(2^x)^2 + 64 \cdot 4^x = 98 \Rightarrow$$
$$4^x = \frac{98}{49} = 2 \Rightarrow \log_4 4^x = \log_4 2 \Rightarrow x = \log_4 2 = \frac{1}{2},$$
which is C.

21. Solve for x: $\sqrt{x^2 - 5x + 40} = 6$

(A) 1 (B) 4 and 1 (C) $\frac{5 \pm \sqrt{76}}{2}$ (D) 3 and 6 (E) NOTA

$$\sqrt{x^2 - 5x + 40} = 6 \Rightarrow x^2 - 5x + 40 = 36 \Rightarrow x^2 - 5x + 4 = 0 \Rightarrow x = \frac{5 \pm \sqrt{25-16}}{2} = \frac{5 \pm 3}{2} = 4{,}1$$
This gives us B.

22. Solve for x: $\log_3 x = \log_x 27$

(A) $\sqrt{3}$ (B) 9 (C) 3 (D) $3\sqrt{3}$ (E) NOTA

$$\log_3 x = \log_x 27 \Rightarrow \log_3 x = 3\log_x 3 = \frac{3}{\log_3 x} \Rightarrow (\log_3 x)^2 = 3 \Rightarrow \log_3 x = \pm\sqrt{3} \Rightarrow x = 3^{\pm\sqrt{3}}.$$
This gives us E.

23. If a, b, and c are rational and $54^a \cdot 50^b \cdot 126^c = 2160$, evaluate $a + b + c$.

(A) 3 (B) 4 (C) 1 (D) 5 (E) NOTA

Using prime factorization, we have
$$2^{a+b+c} \cdot 3^{3a+2c} \cdot 5^{2b} \cdot 7^c = 3^{3a} \cdot 2^a \cdot 5^{2b} \cdot 2^b \cdot 2^c \cdot 3^{2c} \cdot 7^c = (3^3 \cdot 2)^a \cdot (5^2 \cdot 2)^b \cdot (2 \cdot 3^2 \cdot 7)^c =$$
$$54^a \cdot 50^b \cdot 126^c = 2160 = 2^4 \cdot 3^3 \cdot 5$$
so $a + b + c = 4$, which is B.

24. In Problem 23, what is $3a+2c$?

(A) 4 (B) 3 (C) 2 (D) 1 (E) NOTA

From the above factorization, we get $3a + 2c = 3$, which is B.

25. Which of the following is equivalent to $\log_x 25 + \log_5 x - \dfrac{2}{\log_5 x}$?

(A) $\dfrac{1}{\log_{25} x}$ (B) $\log_{25} x - \log_5 x$ (C) $\log_x 5$ (D) $\dfrac{1}{\log_x 5}$ (E) NOTA

Using the equality $\log_c d = \dfrac{\ln d}{\ln c} = \dfrac{1}{\frac{\ln c}{\ln d}} = \dfrac{1}{\log_d c}$, we get

$$\log_x 25 + \log_5 x - \frac{2}{\log_5 x} = 2\log_x 5 + \log_5 x - \frac{2}{\log_5 x} = \frac{2}{\log_5 x} + \log_5 x - \frac{2}{\log_5 x} = \log_5 x = \frac{1}{\log_x 5}$$

This gives us D.

26. Which of the following is equivalent to $\left(2^a 3^{a+2}\right)^{a-2}$?

(A) $\left(\dfrac{6^a}{\left(2^a 3^2\right)}\right)^2$ (B) $\dfrac{\left(2^a 3^a\right)^2}{2^2 3^2}$ (C) $\dfrac{3^{a^2-2a-4} 2^{a^2}}{2^a}$ (D) $3^{a^2-4} 2^{a^2-2a}$ (E) NOTA

$\left(2^a 3^{a+2}\right)^{a-2} = 2^{a(a-2)} 3^{(a+2)(a-2)} = 3^{a^2-4} 2^{a^2-2a}$, which is D.

27. Which of the following is equivalent to $\dfrac{c^{3n+2} b^{2n-1}}{b^{2n+3}\left(c^2\right)^{n+1}}$?

(A) $\dfrac{c^{n-3}}{b^{4-n}}$ (B) $\dfrac{c^n}{b^{2-n}}$ (C) $\dfrac{c^n}{b^4}$ (D) $\dfrac{c^{4+n}}{b^{n-3}}$ (E) NOTA

$\dfrac{c^{3n+2} b^{2n-1}}{b^{2n+3}\left(c^2\right)^{n+1}} = c^{3n+2-2n-2} b^{2n-1-2n-3} = c^n b^{-4} = \dfrac{c^n}{b^4}$, which is C.

28. Evaluate: $7^{\ln(25)}$

(A) $5^{\ln 49}$ (B) $7^{\ln 2 \cdot \ln 5}$ (C) $5^{\ln 7}$ (D) $\ln\left(e^{\ln 7 \cdot \ln(25)}\right)$ (E) NOTA

$7^{\ln(25)} = e^{\ln 7^{\ln 25}} = e^{\ln 25 \ln 7} = e^{2\ln 5 \ln 7} = e^{\ln 5 \ln 49} = e^{\ln 5^{\ln 49}} = 5^{\ln 49}$, which is A.

29. If $2^x = 3$, evaluate $\frac{2\cdot 4^x - 3\cdot 2^x}{4} + \frac{8^x}{4}$.

(A) $\frac{13}{2}$ (B) $\frac{35}{4}$ (C) 9 (D) 7 (E) NOTA

Since $2^x = 3$, $\frac{2\cdot 4^x - 3\cdot 2^x}{4} + \frac{8^x}{4} = \frac{2\cdot\left(2^x\right)^2 - 3\cdot 2^x}{4} + \frac{\left(2^x\right)^3}{4} = \frac{2\cdot 3^2 - 3\cdot 3 + 3^3}{4} = \frac{36}{4} = 9$,
which is C.

30. Solve for x: $a^{\log_{10} a^{x^2}} = m^{\log_{10} m}$

(A) $\sqrt{\log_m a}$ (B) am (C) $\log_m a$ (D) $\log_a m$ (E) NOTA

$$a^{\log_{10} a^{x^2}} = m^{\log_{10} m} \Rightarrow x^2\left(\log_{10} x\right)^2 = x^2 \log_{10} a \log_{10} a = \log_{10} a^{x^2} \log_{10} a = \log_{10} a^{\log_{10} a^{x^2}} =$$
$$\log_{10} m^{\log_{10} m} = \log_{10} m \log_{10} m = \left(\log_{10} m\right)^2 \Rightarrow x^2 = \frac{\left(\log_{10} m\right)^2}{\left(\log_{10} a\right)^2} = \left(\frac{\log_{10} m}{\log_{10} a}\right)^2 = \left(\log_a m\right)^2$$
$$\Rightarrow x = \pm\log_a m$$
Which gives us E. It should also be noted that if a=1, then we cannot make the division. However, if a=1, then if m=1 then all values of x will be solutions. However if $m \neq 1$, then there are no solutions in this case.

31. Solve for x: $\log_7(2x+3) + \log_7(3x-1) = 1$

(A) $\frac{2}{3}$ and -1 (B) $\frac{5}{6}$ and -2 (C) 2 and $-\frac{2}{3}$ (D) 3 and 2 (E) NOTA

$$\log_7\left(6x^2 + 7x - 3\right) = \log_7\left((2x+3)(3x-1)\right) = \log_7(2x+3) + \log_7(3x-1) = 1 \Rightarrow$$
$$6x^2 + 7x - 3 = 7 \Rightarrow 6x^2 + 7x - 10 = 0 \Rightarrow x = \frac{-7 \pm \sqrt{49 + 240}}{12} = \frac{-7 \pm 17}{12} = \frac{5}{6}, -2$$
However, if we allow –2, then we are taking the log of a negative number, and thus we must only consider $\frac{5}{6}$, giving us E.

32. Simplify: $\dfrac{3^x 3^{1-x} 9^x}{27^{\frac{2}{3}x}}$

(A) 9 (B) 3 (C) 3^{x-1} (D) 9^{1-x} (E) NOTA

$\dfrac{3^x 3^{1-x} 9^x}{27^{\frac{2}{3}x}} = \dfrac{3^{x+1-x} 3^{2x}}{3^{3\cdot\frac{2}{3}x}} = \dfrac{3^{1+2x}}{3^{2x}} = 3^{1+2x-2x} = 3^1 = 3$, which is B.

33. Consider the equation $\log_{10}(3x+2) + \dfrac{1}{\log_{2x-1} 10} = 1$, which is satisfied for two rational values of *x*. These two rational values can be expressed as $\dfrac{a}{b}$ and $\dfrac{c}{d}$, where *a* and *b* are relatively prime, as are *c* and *d*. If either $\dfrac{a}{b}$ or $\dfrac{c}{d}$ is negative, treat *a* or *c* as the negative quantity, not *b* or *d*. Evaluate *a* + *b* + *c* + *d*.

(A) 6 (B) 7 (C) 8 (D) 9 (E) NOTA

$\log_{10}((3x+2)(2x-1)) = \log_{10}(3x+2) + \log_{10}(2x-1) = \log_{10}(3x+2) + \dfrac{1}{\log_{2x-1} 10} = 1 \Rightarrow$

$6x^2 + x - 2 = (3x+2)(2x-1) = 10 \Rightarrow 6x^2 + x - 12 = 0 \Rightarrow x = \dfrac{-1 \pm \sqrt{1+288}}{12} = \dfrac{-1 \pm 17}{12} =$

$\dfrac{4}{3}, -\dfrac{3}{2}$

$a + b + c + d = 4 + 3 - 3 + 2 = 6$, which is A.

34. If $\log_2(4x+8) - \log_2(x) \le 3$, what values of *x* are possible.

(A) $x \ge 2$ (B) $x \le 3$ (C) $x \le 2$ (D) $x \ge 3$ (E) NOTA

$\log_2 \dfrac{4x+8}{x} = \log_2(4x+8) - \log_2(x) \le 3 \Rightarrow 4 + \dfrac{8}{x} = \dfrac{4x+8}{x} \le 2^3 = 8 \Rightarrow \dfrac{8}{x} \le 4 \Rightarrow 8 \le 4x \Rightarrow$

$x \ge 2$

This gives us A. We knew that we could multiply by x, since we are taking the logarithm of x
and we can only take logarithms of positive numbers.

35. Which is of the following is equivalent to $\log_4 x^3$?

(A) $\dfrac{2\log_2 x}{3}$ (B) $6\log_2 x$ (C) $\dfrac{3\log_2 x}{2}$ (D) $3\log_{\sqrt{4}} x$ (E) NOTA

$\log_4 x^3 = \dfrac{\log_2 x^3}{\log_2 4} = \dfrac{3\log_2 x}{2}$, which is C.

36. What is the sum of all the positive integral factors of 630?

(A) 900 (B) 1530 (C) 1242 (D) 1872 (E) NOTA

$630 = 2\cdot 3^2 \cdot 5 \cdot 7$, so the sum that we want is
$2\cdot 5\cdot 7\sum_{i=0}^{2} 3^i + 2\cdot 5\sum_{i=0}^{2} 3^i + 2\cdot 7\sum_{i=0}^{2} 3^i + 2\sum_{i=0}^{2} 3^i + 5\cdot 7\sum_{i=0}^{2} 3^i + 5\sum_{i=0}^{2} 3^i + 7\sum_{i=0}^{2} 3^i + \sum_{i=0}^{2} 3^i =$
$144\sum_{i=0}^{2} 3^i = 144(13) = 1872$
This gives us D.

37. If $\log_{10} 5 = a, \log_{10} 7 = b,$ and $\log_{10} 2 = c,$ what is $\log_2 10 + \log_{35} 2$ in terms of a, b, and c.

(A) $\dfrac{a+b+c^2}{ac+bc}$ (B) $\dfrac{ac+b}{ab^2}$ (C) $\dfrac{ac+b}{ab^2c}$ (D) $\dfrac{ab-c}{abc}$ (E) NOTA

$\log_2 10 + \log_{35} 2 = \dfrac{1}{\log_{10} 2} + \dfrac{\log_{10} 2}{\log_{10} 35} = \dfrac{1}{c} + \dfrac{c}{\log_{10} 5 + \log_{10} 7} = \dfrac{1}{c} + \dfrac{c}{a+b} = \dfrac{a+b+c^2}{ac+bc}$, which is A.

38. Which is equivalent to: $\dfrac{r^{-4} - s^{-4}}{r^{-2} - s^{-2}}$?

(A) $r^2 - s^2$ (B) $s^{-2} + r^{-2}$ (C) $\dfrac{r^3 - s^3}{r-s}$ (D) $\dfrac{r^3 - s^3}{r+s}$ (E) NOTA

$\dfrac{r^{-4} - s^{-4}}{r^{-2} - s^{-2}} = \dfrac{\left(r^{-2} + s^{-2}\right)\left(r^{-2} - s^{-2}\right)}{r^{-2} - s^{-2}} = r^{-2} + s^{-2}$, which gives us B.

39. Evaluate: $\dfrac{2i-3}{3i+2}$

(A) -*i* (B) $\dfrac{i}{13}$ (C) i (D) $\dfrac{-i}{13}$ (E) NOTA

$\dfrac{2i-3}{3i+2}\dfrac{3i-2}{3i-2}=\dfrac{6i^2-13i+6}{9i^2-4}=\dfrac{-6-13i+6}{-13}=i$, which is C.

40. Which of the following is equal to: $\log_a\left(\dfrac{b}{c}\right)+\dfrac{1}{\log_b\left(a^3\right)}+\dfrac{1}{\log_c(a)}$

(A) $\log_c\left(\sqrt[3]{a^2b}\right)$ (B) $\log_a\left(\sqrt[3]{b^4}\right)$ (C) $\log_b\left(\sqrt[3]{ac}\right)$ (D) $\log_c\left(\dfrac{\sqrt{ab}}{b}\right)$ (E) NOTA

$\log_a\left(\dfrac{b}{c}\right)+\dfrac{1}{\log_b\left(a^3\right)}+\dfrac{1}{\log_c(a)}=\log_a b-\log_a c+\dfrac{1}{3\log_b a}+\log_a c=\log_a b+\dfrac{\log_a b}{3}=\dfrac{4\log_a b}{3}=$

$\log_a b^{\frac{4}{3}}=\log_a\sqrt[3]{b^4}$

This gives us B.