

# Solutions - Equations & Inequalities

## Alpha Page 1

1.  $Bart \geq Arthur + \frac{100}{100} Arthur \geq 2 \cdot Arthur$

$\Rightarrow$   $Chris \geq Bart + \frac{200}{100} Bart = 4 \cdot Bart = 8 \cdot Arthur$

$Donald \geq Chris + \frac{50}{100} Chris = 1.5 Chris = 12 \cdot Arthur$

$Arthur \leq \frac{Donald}{12} \leq 350,000$  A

2.  $x^2 - 7x + 17 > 5$

$x^2 - 7x + 12 = 0$

$(x-4)(x-3) = 0$

Case I:  $x = 5$ ,  $x^2 - 7x + 17 = 7$ ,  $7 > 5$

Case II:  $x = 3.5$ ,  $x^2 - 7x + 17 = 4.75$ ,  $4.75 < 5$

Case III:  $x = 2$ ,  $x^2 - 7x + 17 = 7$ ,  $7 > 5$

$x < 3$  or  $x > 4$ , D

3.  $x^3 = -27$

$x = -3, -3 \angle 120, -3 \angle -120$

$x = -3, -3(\cos 120 + i \sin 120), -3(\cos 120 + i \sin -120)$

$x = -3, \frac{3}{2} - \frac{3\sqrt{3}}{2}i, \frac{3}{2} + \frac{3\sqrt{3}}{2}i$  B

4.  $\cot x = 1$  when  $\tan x = 1$

$\tan x = 1$  at  $45^\circ, 225^\circ$  A

5.  $|\sec x| = 1$  when  $|\cos x| = 1$  D  $0^\circ, 180^\circ$

6.  $x = \frac{8}{3 + 5f^{-1}(x)}$   $3x + 5x f^{-1}(x) = 8$

$f^{-1}(x) = \frac{8-3x}{5x}$  A

7.  $f(14) = \frac{2(14)^2}{3} = \frac{392}{3}$

$g(d) = g\left(\frac{392}{3}\right) = 3\left(\frac{392}{3}\right) + 2 = 394$  C

# Equations and Inequalities

## Solutions - Alpha page 2

$$8. \quad 2^{f-4} 2^{4f+16} = 2^{9f-9}$$

$$5f + 12 = 9f - 9$$

$$-4f = -21 \quad f = \frac{21}{4} \quad \underline{D}$$

$$9. \quad 2^{x-2} 3^{x-2} 2^{3y+6} 3^{y+2} = 2^6 3^{10}$$

$$x + 3y + 4 = 6$$

$$x + y = 10$$

$$-x - y = -10$$

$$2y = -8$$

$$y = -4 \quad x = 14$$

B

$$10. \quad 3x^3 + 6x^2 + 4x + 1 = 0$$

$$\begin{array}{r}
 \phantom{x+1} \quad 3x^2 + 3x + 1 \\
 x+1 \overline{) 3x^3 + 6x^2 + 4x + 1} \\
 \underline{-3x^3 - 3x^2} \phantom{+ 1} \\
 3x^2 + 4x \phantom{+ 1} \\
 \underline{-3x^2 - 3x} \phantom{+ 1} \\
 x + 1 \phantom{+ 1} \\
 \underline{-x - 1} \\
 0
 \end{array}$$

First solution, -1  
can be found by  
trial and error or  
with a calculator.

$$\text{Sum of imaginary roots} = \frac{-b}{a} = \frac{-3}{3} = -1 \quad \underline{D}$$

$$11. \quad x^5 - 2x^2 + 3x + 5 = f(x)$$

$$\frac{-p}{E} = \frac{-3}{5} \quad \underline{A}$$

$$12. \quad f(x-3) = 3x^2 - 5x - 2$$

$$\text{Let } u = x-3 \quad x = u+3$$

$$f(u) = 3(u+3)^2 - 5(u+3) - 2$$

$$f(u) = 3u^2 + 18u + 27 - 5u - 15 - 2$$

$$f(u) = 3u^2 + 13u + 10$$

$$f(x) = 3x^2 + 13x + 10$$

$$\frac{f(x)}{5} = \frac{3x^2}{5} + \frac{13x}{5} + 2$$

C

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$$13. \frac{3x+9}{x^2+7x+10} = \frac{A}{x+2} + \frac{B}{x+5}$$

$$3x+9 = A(x+5) + B(x+2)$$

$$3 = A+B \quad \underline{C}$$

$$14. \frac{25x-11}{x^2-2x+1} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$$

$$25x-11 = A(x-1) + B$$

$$A=25 \quad \underline{B} \quad -A+B=-11$$

$$B=14 \quad \underline{B}$$

$$15. \quad x + y + z = 1$$

$$3x - 2y + 3z = 5$$

$$2x + 2y + 2z = 2$$

$$5x \quad + 5z = 7$$

$$x = -z + \frac{7}{5} \quad \underline{B}$$

$$16. \quad x + 3y + 4z = -5$$

$$-x + y - 2z = 5$$

$$-5x - y - z = 1$$

$$\left[ \begin{array}{ccc|c} 1 & 3 & 4 & -5 \\ -1 & 1 & -2 & 5 \\ -5 & -1 & -1 & 1 \end{array} \right] = \left[ \begin{array}{ccc|c} 1 & 3 & 4 & -5 \\ 0 & 4 & 2 & 0 \\ 0 & 14 & 19 & -24 \end{array} \right] \begin{array}{l} R_1 + R_2 \\ 5R_1 + R_3 \end{array}$$

$$= \left[ \begin{array}{ccc|c} 1 & 3 & 4 & -5 \\ 0 & 1 & \frac{1}{2} & 0 \\ 0 & 14 & 19 & -24 \end{array} \right] = \left[ \begin{array}{ccc|c} 1 & 0 & \frac{5}{2} & -5 \\ 0 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 12 & -24 \end{array} \right] \begin{array}{l} R_1 - 3R_2 \\ R_3 - 14R_2 \end{array}$$

$$= \left[ \begin{array}{ccc|c} 1 & 0 & \frac{5}{2} & -5 \\ 0 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & -2 \end{array} \right] = \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array} \right] \begin{array}{l} R_1 - \frac{5}{2}R_3 \\ R_2 - \frac{1}{2}R_3 \end{array}$$

$$x=0 \quad y=1 \quad z=-2 \quad \underline{B}$$

Solutions - Equations & Inequalities  
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17.  $\log_2(2^{2/3} x^{1/3}) + \log_2(2x^4) = -47/3$   
 $\log_2(2^{5/3} x^{13/3}) = -47/3$   
 $2^{5/3} x^{13/3} = 2^{-47/3}$   
 $x^{13/3} = 2^{-52/3} \quad x = 2^{-4} = \frac{1}{16} \quad \underline{E}$   
 $\sqrt{x} - x - 10x^2 = \frac{1}{4} - \frac{1}{16} - 10\left(\frac{1}{256}\right) = \frac{38}{256} = \frac{19}{128} \quad \underline{\cancel{X}}$

18.  $\frac{\log x}{\log 10} = \frac{\log 27}{\log 9} \quad \log x = \frac{3}{2} \quad x = 10\sqrt{10}$   
 $x^3 - 50x = 10000\sqrt{10} - 500\sqrt{10} = 9500\sqrt{10} \quad \underline{A}$

19. I want  $\lfloor \log_2 N \rfloor$  and  $\log_2 N$  the greatest possible distance apart. This is when  $N = 3^x - 1$  where  $x$  is any positive integer. In this problem,  $x$  is constrained to be 6 or less. The lowest value is  $5 - \log_3 728 \quad \underline{B}$

20.  $\frac{x-2}{-3x-2} \geq -3$   
 Critical point at  $x = -2/3$   
 $x-2 \geq 9x+6$   
 $x \leq -1$  Critical point at  $x = -1$   
 case I:  $x = -2 \quad \frac{x-2}{-3x-2} = \frac{-4}{4} = -1 \geq -3$   
 case II:  $x = -3/4 \quad \frac{x-2}{-3x-2} = \frac{-11/4}{1/4} = -11 \leq -3$   
 $x \leq -1$  or  $x \geq -2/3 \quad \underline{E}$

21. Bill = Mary + 5  
 Bill + 3 = 7(Mary - 10)  
 -Bill = -Mary - 5  
 3 = 6·Mary - 75  
 Mary = 12 Bill = 17 A

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22.  $24 - 448q + 11 = 5$   
 $q = \frac{15}{224}$  E

23.  $4x^2 + 30x + c = 0$

Case I:  $2 \left( \frac{-30 + \sqrt{900 - 16c}}{8} \right) = \frac{-30 - \sqrt{900 - 16c}}{8}$

$-30 = -3\sqrt{900 - 16c}$

$10 = \sqrt{900 - 16c}$

$c = 50$  C

24.  $A + B = 3$   
 $A + D = 2$   
 $D + E = 2$   
 $A + E = 2$

$A + D = 2$   
 $-E - D = -2$   
 $A - E = 0$   
 $A + E = 2$

$2A = 2$

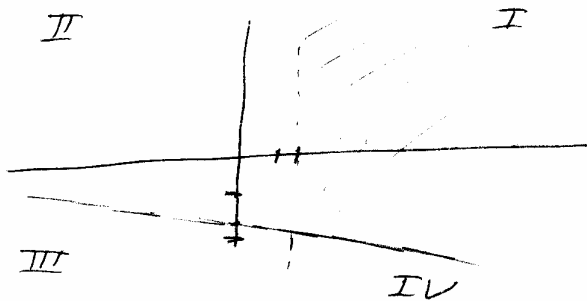
$A = 1 \quad B = 2 \quad D = 1 \quad E = 1 \quad C = 0$

C

25.  $m = 4k^{-1}(m) - 1$

$\frac{m+1}{4} = k^{-1}(m)$  B

26.  $-3y < x + 5$   
 $y \geq -\frac{1}{3}x - \frac{5}{3}$       $x \geq \frac{4}{3}$



B

27. C ellipse

Solutions - Equations & Inequalities  
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28.  $f(7) = \frac{35}{25} = \frac{7}{5}$      $g(\frac{7}{5}) = \frac{63}{5} - \frac{10}{5} = \frac{53}{5}$     B

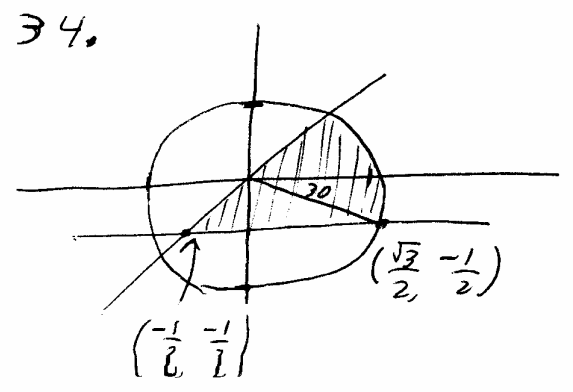
29.  $9w - 39 < 94$   
 $9w < 133$   
 $w < 14\frac{7}{9}$     D

30.  $t+1 < -t-4$      $-t-4 < 3t+17$   
 $2t < -5$      $-21 < 4t$   
 $t < -\frac{5}{2}$      $t > -\frac{21}{4}$     D

31.  $x = \frac{a}{1+r}$      $y = \frac{a}{1-r^2}$      $z = \frac{a}{1-r^3}$   
 $y = \frac{a}{(1+r)(1-r)}$      $z = \frac{a}{(1-r)(1+r+r^2)}$   
 $\frac{a}{(1+r)(1+r^2)} = \frac{a(1-r)(1+r)}{(1-r)(1+r)(1+r+r^2)} = \frac{z \cdot x}{y}$     B

32.  $\cot \theta \geq 1$ ,  $\tan \theta \leq 1$      $0 \leq \theta \leq \frac{\pi}{4}$     D

33. period of  $\sin(x)$  is  $2\pi$   
 period of  $\sin(2x)$  is  $\pi$   
 period of  $\sin(3x)$  is  $\frac{2\pi}{3}$   
 period of  $\sin(4x)$  is  $\frac{\pi}{2}$   
 period of  $\sin(5x)$  is  $\frac{2\pi}{5}$   
 period of function is  $\text{LCM}(2\pi, \pi, \frac{2\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{5}) = 2\pi$   
B



Area of 75° wedge =  $\frac{75}{360} \cdot \pi = \frac{5\pi}{24}$   
 Area of triangle =  $\frac{1}{2} b h$   
 $= \frac{1}{2} (\frac{1}{2} + \frac{\sqrt{3}}{2}) (\frac{1}{2}) = \frac{1}{8} + \frac{\sqrt{3}}{8}$   
 Total Area =  $\frac{5\pi}{24} + \frac{1}{8} + \frac{\sqrt{3}}{8}$     C

# Alpha Equations & Inequalities

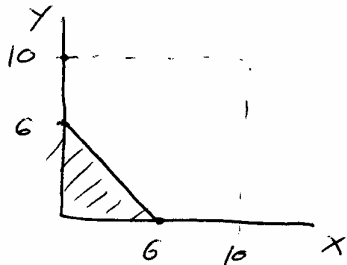
## Solutions page 7

35.  $y(x) = \text{Arcsin}(\ln(x))$

domain of Arcsin is  $-1 \leq x \leq 1$

$-1 \leq \ln(x) \leq 1 \quad \frac{1}{e} \leq x \leq e \quad \underline{E}$

36.



probability =  $\frac{36}{100} = \frac{9}{50}$

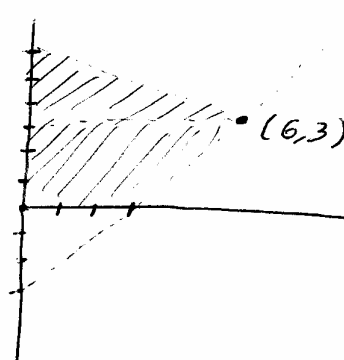
A

37.

$|(4-3i)| |(5+12i)| |(8-15i)|$

$= (5)(13)(17) = 1105 \quad \underline{A}$

38.



$y \geq x - 3$

$y \leq -\frac{x}{2} + 6$

Area =  $\frac{6 \cdot 3}{2} + \frac{1}{2}(6+3)(3) = 9 + \frac{27}{2} = \frac{45}{2} \quad \underline{C}$

39.

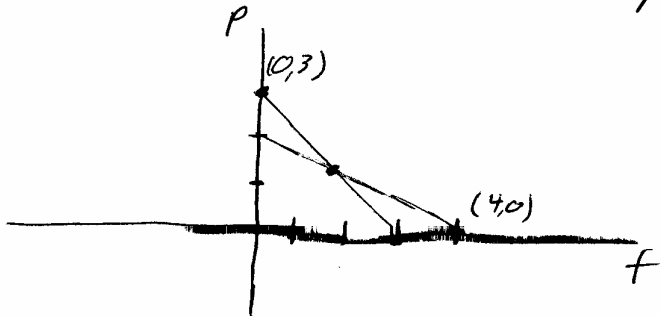
Let  $f$  = servings of french fries

Let  $p$  = servings of pizza

$600f + 600p \geq 1800$

$2f + 4p \geq 8$

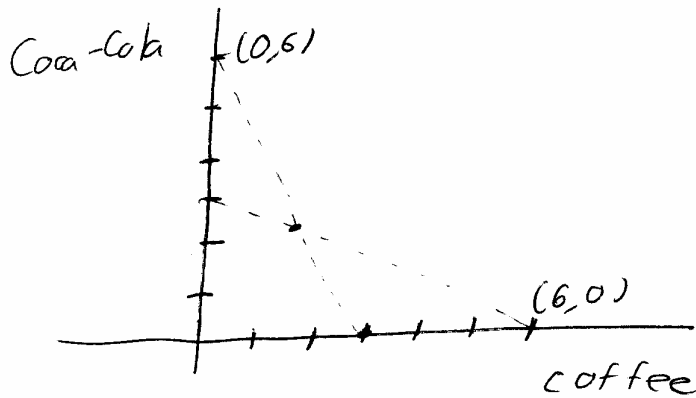
minimize  $f + 1.5p$



corner 1:  $(f, p) = (4, 0) = \$4.00$   
 corner 2:  $(f, p) = (2, 1) = \$3.50$   
 corner 3:  $(f, p) = (0, 3) = \$4.50$

D

$$\begin{aligned}
 25x + 50y &\geq 150 \\
 10x + 5y &\geq 30 \\
 \text{minimize } x + 1.5y
 \end{aligned}$$



$$x + 2y = 6$$

$$2x + y = 6$$

$$-3y = -6$$

$$y = 2$$

$$x = 2$$

3 corners:

$$\begin{aligned}
 (6, 0) &= \$6.00 \\
 (0, 6) &= \$9.00 \\
 (2, 2) &= \$5.00
 \end{aligned}$$

A