Mu Alpha Theta National Convention: Denver, 2001
Individual Test – Mu Division

1. Find the sum of all x that satisfy the equation: $x^2 - x - 5 = 4x - 3x^2 - \frac{7}{2}$

(A) $\frac{8}{3}$ (B) $\frac{5}{4}$ (C) $\frac{10}{3}$ (D) $-\frac{3}{8}$ (E) NOTA

2. Simplify: $\frac{3\sqrt{96}}{4} + 2\sqrt[3]{40} + \frac{\sqrt{216}}{3} + \frac{2\sqrt[3]{135}}{3}$

(A) $5\sqrt{6} + 6\sqrt[3]{5}$ (B) $6(\sqrt{6} + \sqrt[3]{5})$ (C) $7\sqrt{6} + 4\sqrt[3]{5}$ (D) $9\sqrt{6} + 7\sqrt[3]{5}$ (E) NOTA

3. The points (6, 3), (-1, 2), and (9, k) are collinear, and $k = \frac{m}{n}$, where m and n are relatively prime positive integers. Find the value of $\sqrt{m^2 + n^2}$.

(A) 41 (B) 25 (C) 13 (D) 10 (E) NOTA

4. Let $i = \sqrt{-1}$. Find the reciprocal of 8 + 15i.

(A) $\frac{8-15i}{289}$ (B) $\frac{-8-15i}{161}$ (C) $\frac{-8-15i}{289}$ (D) $\frac{8-15i}{161}$ (E) NOTA

5. Two fair, six-sided dice are thrown. What's the probability that the sum of the numbers shown is 8?

(A) $\frac{1}{12}$ (B) $\frac{1}{6}$ (C) $\frac{5}{36}$ (D) $\frac{1}{9}$ (E) NOTA

6. Given that $\log(x-3) = 3$, what is the value of x when its digits are reversed?

(A) 799 (B) 3001 (C) 678 (D) 1001 (E) NOTA

7. U is the units digit of 71182^{111082}. W is the number of positive integral divisors of 221. What is the value of the product UW?

(A) 8 (B) 2 (C) 16 (D) 32 (E) NOTA

8. Given that $-\frac{\pi}{2} < \text{Arctan}(t) < \frac{\pi}{2}$ for all t, evaluate $\sin\left(\text{Arctan}\left(\frac{33}{56}\right)\right)$.

(A) $\frac{56}{33}$ (B) $\frac{56}{65}$ (C) $\frac{33}{56}$ (D) $\frac{33}{65}$ (E) NOTA

9. Given that $y = f(x) = \frac{2x^2 + 5x - 7}{5x^2 - 2x - 3}$. How many asymptotes does the graph of f have?

(A) 0 (B) 1 (C) 2 (D) 3 (E) NOTA

10. The value of $\lim_{c \to 0} \frac{(x+c)^4 - x^4}{c}$ when $x = 2$ is

(A) 16 (B) 32 (C) 8 (D) 64 (E) NOTA

11. Find $x^2 + y^2 + z^2$, where x, y, and z are the solutions of the system $\begin{matrix} 2x - y + z = 3 \\ 5x + 2y + 2z = 15 \\ -3x + 4y - 3z = -4 \end{matrix}$.

(A) 25 (B) $\frac{501}{25}$ (C) 21 (D) 14 (E) NOTA

12. The position of a particle is given by $f(t) = \begin{cases} t^2 & 0 \le t \le 3 \\ t + 6 & 3 < t \le 10 \end{cases}$. What is the average speed of the particle on the interval $2 \le t \le 8$?

(A) 1 (B) $\frac{5}{3}$ (C) $\frac{17}{6}$ (D) $-\frac{1}{2}$ (E) NOTA

13. What is the sum of the real zeros of the function $f(x) = x^4 - 3x^3 + 3x^2 - 3x + 2$?

(A) 2 (B) $-\frac{3}{2}$ (C) 3 (D) -2 (E) NOTA

14. As x increases at $x = e^4$, the graph of $y = x \ln x - x$ is

(A) decreasing & concave down (B) decreasing & concave up
(C) increasing & concave down (D) increasing & concave up (E) NOTA

15. Let $\underline{A} = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix}$. What is the value of $|\underline{A}|$?

(A) -18 (B) 50 (C) -1712 (D) -128 (E) NOTA

16. Let R be the region satisfying $y < x^2 + 1$, $y > x - 1$, $x < 1$, and $x > 0$. What is the area of R?

(A) $\frac{11}{6}$ (B) $\frac{3}{2}$ (C) $\frac{5}{6}$ (D) 2 (E) NOTA

17. What is the locus of the centers of all circles tangent to two distinct parallel lines?

(A) parabola (B) hyperbola (C) circle (D) line (E) NOTA

18. Which of the following is/are always true:

I. If $\lim_{x \to c} f(x)$ and $\lim_{x \to c} g(x)$ do not exist,
then $\lim_{x \to c} \left(f(x) + g(x) \right)$ does not exist either.

II. If $f'(x)$ is defined at $x = c$, then $f(x)$ is also defined there.

III. If $\int_a^b f(x)\, dx > 0$, then $f(x) > 0$ on $a \le x \le b$.

(A) I & III only (B) I & II only (C) II only (D) II & III only (E) NOTA

19. Equilateral triangle *ABC* with a side-length of 2 is inscribed in 30°-60°-90° triangle *DEF* such that each of its vertices is on a different side of *DEF*, and none of them is coincident with a vertex of *DEF*. If one side of the equilateral triangle is parallel to the shortest side of *DEF*, determine the area of *DEF*.

(A) $\frac{9\sqrt{3}}{2}$ (B) $\frac{3+3\sqrt{3}}{2}$ (C) $\sqrt{3}$ (D) $2 + \frac{2\sqrt{3}}{3}$ (E) NOTA

20. Evaluate: $\int_0^{\pi/4} \frac{\cos 2x}{|\cos x + \sin x|}\, dx$

(A) $1 + \sqrt{2}$ (B) 1 (C) $\sin\frac{\pi}{4} + 1$ (D) $\sqrt{2} - 1$ (E) NOTA

21. A sequence is defined recursively as follows: $a_1 = 5$, $a_2 = 9$, $a_n = \dfrac{2a_{n-1} + a_{n-2}}{3}$.
Evaluate: $\lim\limits_{n\to\infty}(a_n)$.

(A) $\frac{25}{3}$ (B) $\frac{33}{4}$ (C) $\frac{71}{9}$ (D) $\frac{127}{16}$ (E) NOTA

22. For the fourth degree polynomial $P(x)$ such that $P(0) = P(1)$, $P'(0) = P''(0)$, and $P'''(1) = 54$, find the ratio of the coefficient of the *x*-term to the coefficient of the x^2-term.

(A) 6 (B) $\frac{1}{9}$ (C) $-\frac{1}{4}$ (D) 2 (E) NOTA

23. In triangle *ABC*, *BC* = 6, *AC* = 8, and *AB* = 10. *CD* is the altitude to *AB* and *CE* is the median to *AB*. Find the area of triangle *CDE*.

(A) $\frac{91}{20}$ (B) 12 (C) $\frac{84}{25}$ (D) $\frac{12\sqrt{5}}{25}$ (E) NOTA

24. Solve the system of congruences: $\begin{matrix} 8x \equiv 24 \pmod{7} \\ 2x \equiv 18 \pmod{13} \end{matrix}$ where *x* is an arbitrary integer.

(A) $x \equiv 2 \bmod 91$ (B) $x \equiv 3 \bmod 91$ (C) $x \equiv 87 \bmod 91$ (D) $x \equiv 88 \bmod 91$ (E) NOTA

25. Let $f(x) = |x-1| + |x-2|$, $I = \int_0^3 f(x)\, dx$, *M* = the minimum value of *f*, *N* = $f'(x)$ for $x < -4$, and *C* = the value of $f''(4)$. Evaluate: $\dfrac{M^2 - N^2 + IC}{2}$.

(A) $-\frac{3}{2}$ (B) $-\frac{5}{2}$ (C) $\frac{3}{2}$ (D) $\frac{5}{2}$ (E) NOTA

26. Let $\sum_{n=4}^{8} \binom{n}{4} = \binom{x}{y}$, where *x* and *y* are positive integers. Find the smallest possible value of $x + y$.

(A) 13 (B) 14 (C) 15 (D) 127 (E) NOTA

27. Evaluate: $\int_1^e \left(x+\frac{2}{x}\right)dx$

(A) $\frac{e^2-1}{2}$ (B) $\frac{e^2+3}{2}$ (C) $\frac{e^2+4e-3}{2}$ (D) $\frac{3e^2+4e}{2}$ (E) NOTA

28. What is the volume of the solid formed by revolving the region satisfying $y < x\ln x$, $x > e^4$, and $x < e^5$, and $y > 0$ about the y-axis?

(A) $\frac{8\pi e^{15}}{3} - 2\pi e^{12}$ (B) $\frac{197\pi e^{15} - 122\pi e^{12}}{27}$
(C) $\frac{9\pi e^{10} - 7\pi e^{8}}{2}$ (D) $\frac{28\pi e^{15} - 22\pi e^{12}}{9}$ (E) NOTA

29. Let a and b be the legs of a right triangle with hypotenuse 2. Determine the value of $a^8 + 4a^6b^2 + 6a^4b^4 + 4a^2b^6 + b^8$.

(A) 4 (B) 16 (C) 64 (D) 256 (E) NOTA

30. The power series $P(x) = \sum_{n=3}^{\infty} \frac{n^2(2x-1)^n}{2^n}$ converges on the interval $a < x < b$, where a and b are real numbers; otherwise P diverges. What is $|b - a|$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) NOTA

31. Let $i = \sqrt{-1}$. Find $x^2 + 2xy + y^2$, where $\frac{x}{3-5i} + \frac{y}{2+7i} = \frac{5+2i}{41+11i}$ and x and y are both real numbers.

(A) 16 (B) 4 (C) 9 (D) 1 (E) NOTA

32. Suppose $\int_{12}^{14} f(x)\,dx \ge \frac{3}{4}$ and $\int_{12}^{14} g(x)\,dx \ge \frac{7}{2}$, where f and g are continuous, integrable functions of x. Which of the following is true about $I = \int_6^7 \left(f(2x) + g(2x)\right) dx$?

(A) $I \ge \frac{17}{8}$ (B) $I \le \frac{11}{4}$ (C) $I \ge \frac{17}{4}$ (D) $I \le \frac{11}{8}$ (E) NOTA

33. C is a curve defined by the parametric equations $x = t^2$ and $y = t^3 - 3t$. What is the equation of the tangent line to this curve at (4, 2)?

(A) $y = 15x - 58$ (B) $y = 2$ (C) $y = x - 2$ (D) $y = \frac{9}{4}x - 7$ (E) NOTA

34. Let a be a positive integer. If $a^n - 1$ is prime for <u>some</u> integers $n \geq 3$, what is the sum of all possible values of a?

(A) 2 (B) 5 (C) 7 (D) 10 (E) NOTA

35. Which of the following statements are always true?

I. If $\sum_{n=1}^{\infty} |b_n|$ converges, then $\sum_{n=1}^{\infty} (-1)^n b_n$ is absolutely convergent.

II. If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n b_n$ converges.

III. If $a_n > 0$ and $\lim_{n \to \infty} 2^n a_n = L > 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

(A) I & II only (B) I, II, & III (C) I & III only (D) I only (E) NOTA

36. Let $f(x) = x^3 + 2x - 1$ and g be the inverse function of f, both considered to have domains and ranges which are subsets of the real numbers. Find the value of $|g''(2)|$.

(A) $\frac{6}{125}$ (B) $\frac{1}{6}$ (C) $\frac{5}{144}$ (D) 12 (E) NOTA

37. Suppose A, B, and C are sets. Define X^c as the complement of a set X. Which of the following is equivalent to $(C \cup A^c) \cap (C \cup B^c)$?

(A) $C \cup A^c \cup B^c$ (B) $A^c \cup B^c$
(C) $C \cup (A \cup B)^c$ (D) $\varnothing$ (E) NOTA

38. Let $f(x) = 3x^2 - 2x + 1$, $g(x) = x^2 - 2x + 3$, and $h(x) = f(g(x))$. Evaluate $h'(0)$.

(A) -32 (B) -8 (C) 0 (D) 4 (E) NOTA

39. Given that $\lim_{x\to 2}\left(\frac{4x^2-16x+16}{3x^3-9x^2+12}\right)=\frac{m}{n}$, where *m* and *n* are relatively prime natural numbers, determine the sum of *m* and *n*.

(A) 10 (B) 11 (C) 12 (D) 13 (E) NOTA

40. For all real numbers *x*, $f(\cos^2 x)=\cos 2x$. Evaluate $\sum_{n=1}^{16} f\left(\frac{n}{32}\right)$.

(A) -8 (B) $-\frac{15}{2}$ (C) 8 (D) $\frac{1}{2}$ (E) NOTA

41. Evaluate: $\int_0^{\sqrt{6}/3}\left(x\sqrt{2-3x^2}\right)dx$

(A) $\frac{4}{3}$ (B) $\frac{\sqrt{15}}{6}$ (C) $\frac{2\sqrt{2}}{9}$ (D) $\frac{4\sqrt{6}}{5}$ (E) NOTA

42. Let $x+\frac{1}{x}=1$ and *a*, *b*, and *c* are distinct positive integers for which $x^a+\frac{1}{x^a}+x^b+\frac{1}{x^b}+x^c+\frac{1}{x^c}=0$. Find the minimum value of *a* + *b* + *c*.

(A) 8 (B) 9 (C) 12 (D) 13 (E) NOTA

43. Determine $\frac{dy}{dx}$ for the relationship $\frac{3y+2x}{4x+7y}=2y+3x$.

(A) $\frac{12x+25y-3}{7-5y-18x}$ (B) $\frac{24x+29y-2}{3-28y-29x}$

(C) $\frac{3x-2y+3}{7+15y+9x}$ (D) $\frac{18x-y+2}{3+4y+6x}$ (E) NOTA

44. Sequence *S* is a finite, integer-valued, geometric sequence with first term 1 and last term 1024. Furthermore, *S* has an even number of terms and more than two terms. What is the sum of the terms of *S*?

(A) 1057 (B) 1365 (C) 2037 (D) 2047 (E) NOTA

45. Evaluate: $\int \left(\sqrt{1-x^2}\right) dx$

(A) $\dfrac{\arcsin(x)-\sqrt{1-x^2}+C}{2}$ (B) $\dfrac{\arcsin(x)+\sqrt{1-x^2}+C}{2}$

(C) $\dfrac{\arcsin(x)-x\sqrt{1-x^2}+C}{2}$ (D) $\dfrac{\arcsin(x)+x\sqrt{1-x^2}+C}{2}$ (E) NOTA

46. How many ways are there to arrange the letters in the "word" MATHISFUN so that at least one of the words MATH, IS, or FUN appears?

(A) 45218 (B) 45219 (C) 45220 (D) 45221 (E) NOTA

47. In ΔABC, D is the foot of angle bisector of $\angle ACB$. If $AC = 5$, $AD = 3$, and $DB = 6$, what is the length of CD?

(A) $4\sqrt{2}$ (B) $2\sqrt{7}$ (C) $\dfrac{18}{5}$ (D) $\dfrac{2\sqrt{3}}{3}$ (E) NOTA

48. For how many real values of x will $\begin{bmatrix} x-2 & x+5 \\ 2x & x-3 \end{bmatrix}$ not have an inverse?

(A) 0 (B) 1 (C) 2 (D) 3 (E) NOTA

49. AB and CD are perpendicular diameters of circle O. CM is a chord that intersects AB at E. If $CE = 4$ and $EM = 3$, what is the area of O?

(A) 14π (B) 15π (C) 16π (D) 17π (E) NOTA

50. The volume of a regular tetrahedron is changing at a rate of 6 cubic feet per second (the edge lengths are increasing uniformly, so that the figure remains a tetrahedron). How fast is the edge length changing (in feet per second) when the area of one of the faces is $\sqrt{3}$ square feet?

(A) $3\sqrt{2}$ ft/sec (B) $5\sqrt{3}$ ft/sec (C) $8\sqrt{2}$ ft/sec (D) $6\sqrt{2}$ ft/sec (E) NOTA