

Mu Alpha Theta National Convention: Denver, 2001
Colorado Bowl – Theta Division

1. How many times do the graphs of the equations $y = 2x^2 - 3x + 5$ and $y = 3x^3 + 10x^2 - 8x - 1$ intersect in the Cartesian plane?
(A) 3 (B) 2 (C) 1 (D) 0 (E) NOTA

2. What is the sum of the terms in the infinite geometric sequence with first term 24 and common ratio $\frac{1}{4}$?
(A) 30 (B) 32 (C) 36 (D) 48 (E) NOTA

3. Solve for x : $\log_{12} 48 - \log_{12} x = 2$
(A) $\ln(3)$ (B) $\frac{1}{3}$ (C) 1 (D) 3 (E) NOTA

4. Two circles of equal radius intersect in such a manner that their points of intersection are 8 centimeters apart, and each circle passes through the other's center. What is the common radius of these circles, in centimeters?
(A) $\frac{8\sqrt{3}}{3}$ cm (B) 8 cm (C) $\frac{4\sqrt{3}}{3}$ cm (D) $2\sqrt{3}$ cm (E) NOTA

5. What is the volume, in cubic meters, of a right rectangular pyramid with height 12 meters, base width 32 meters, and base length 18 meters?
(A) 6912 m^3 (B) 4096 m^3 (C) 2304 m^3 (D) 1080 m^3 (E) NOTA

6. The sum of the digits in a two-digit number is 10. If the digits are reversed, the resulting number is 54 less than the original number. What is the largest prime factor of the original number?
(A) 2 (B) 13 (C) 73 (D) 41 (E) NOTA

7. If it takes three hours for Bill to paint a room, and 100 minutes for Brian to do the same, how many minutes will it take them to paint five rooms, working together? Note: all rooms discussed in this problem are the same size as one another.
(A) 360 min. (B) 350 min. (C) $\frac{2250}{7}$ min. (D) 340 min. (E) NOTA

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8. The length of a rectangle is 7 centimeters more than the width. If the length is increased by 3 centimeters and the width by 4 centimeters, the area is increased by 44 square centimeters. What is the width of the original rectangle, in centimeters?

(A) $\frac{4}{7}$ cm (B) 4 cm (C) 7 cm (D) $\frac{7}{4}$ cm (E) NOTA

9. You have 8 liters of a 15% salt-water solution. How many more liters of water must you add to get it to be a 10% salt-water solution?

(A) 4 liters (B) 5 liters (C) 6 liters (D) $\frac{36}{5}$ liters (E) NOTA

10. Find the equation of a parabola with focus at $(-2, 3)$ and directrix at $y = -1$.

(A) $y = x^2 + 4x + 5$ (B) $y = \frac{1}{8}(x+2)^2 + 1$
(C) $y = \frac{1}{2}(x+2)^2 + 1$ (D) $y = \frac{1}{2}(x-2)^2 + 1$ (E) NOTA

11. The graph of $\frac{x^2}{6} - \frac{y^2}{16} = 1$ is a(n)

(A) hyperbola (B) ellipse (C) parabola (D) lemniscate (E) NOTA

12. Find the sum of the following complex numbers: $-1 - 3i$, $2i + 4$, $5i$, $1 - 6i$

(A) $12 + 4i$ (B) $-3 + i$ (C) $4 - 2i$ (D) $-9 - 4i$ (E) NOTA

13. Evaluate: $\frac{1+2i}{4-3i}$

(A) $\frac{-4+i}{5}$ (B) $\frac{6-8i}{5}$ (C) $\frac{-4+i}{25}$ (D) $\frac{-2+11i}{25}$ (E) NOTA

14. Evaluate: $\sum_{n=1}^{10} (-2i)^n$

(A) $246 - 182i$ (B) $-820 - 410i$ (C) $-1023 + 511i$ (D) $-1024 + 512i$ (E) NOTA

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15. Circle A is inscribed within square B. Circle C is circumscribed about square B. What is the ratio between the area of circle C and the area of circle A?

- (A) 4:3 (B) $\sqrt{2}:1$ (C) $\sqrt{3}:1$ (D) 2:1 (E) NOTA

16. Find the sum of all x for which $5 + \sqrt{x^2 - 3x - 14} = 7$.

- (A) -9 (B) -6 (C) 0 (D) 3 (E) NOTA

17. Solve for y given that x and y are both rational. $12^x \cdot 18^y = 648$

- (A) 1 (B) 2 (C) $-\frac{4}{3}$ (D) $\frac{5}{3}$ (E) NOTA

18. Solve for x : $\frac{3x+7}{2-2x} \leq 5$

- (A) $x \leq \frac{3}{13}$ or $x > 1$ (B) $x \leq \frac{3}{13}$
(C) $\frac{3}{13} \leq x < 1$ (D) $x \geq \frac{3}{13}$ (E) NOTA

19. Solve for t : $\frac{t-1}{2} \geq 5 - 2t > 3t + 1$

- (A) $-\frac{3}{5} \leq t < \frac{4}{5}$ (B) $-\frac{3}{5} \leq t < \frac{11}{5}$ (C) $\frac{4}{5} < t < \frac{11}{5}$ (D) $t \leq -\frac{3}{5}$ (E) NOTA

20. Given that $g(d) = d^2 - 5$ and $f(d) = 8 - 3d$, what is $g(f(7))$?

- (A) 164 (B) 191 (C) 220 (D) 251 (E) NOTA

21. What is the midpoint of the line segment connecting $(3, 14)$ and $(-8, 20)$?

- (A) $(-5, 17)$ (B) $(-5, 6)$ (C) $\left(-\frac{5}{2}, 17\right)$ (D) $\left(-\frac{5}{2}, 6\right)$ (E) NOTA

22. How many 6-centimeter by 8-centimeter tiles would it take to completely tile the floor of a 30-meter by 39-meter room, assuming only whole tiles may be used?

- (A) 240,000 (B) 243,750 (C) 256,000 (D) 258,250 (E) NOTA

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23. What is the ratio of the volume of a cylinder of height 18 and base radius 6 to the volume of a sphere of radius 12?

- (A) $\frac{1}{8}$ (B) $\frac{3}{16}$ (C) $\frac{1}{4}$ (D) $\frac{9}{32}$ (E) NOTA

24. Two circles have radii of 8 and 23 centimeters, and their centers are 25 centimeters apart. What is the length, in centimeters, of their common external tangent?

- (A) $5\sqrt{6}$ (B) $10\sqrt{2}$ (C) 20 (D) $10\sqrt{3}$ (E) NOTA

25. A cow is tethered to an external corner of a rectangular barn. If the length of the tether is twenty meters, and the dimensions of the barn are fifteen by twenty meters, what is the total area, in square meters, that the cow can graze?

- (A) 315π (B) 320π (C) 325π (D) $\frac{1225\pi}{4}$ (E) NOTA

26. Solve for x : $4^{3x+2} = 128$

- (A) -1 (B) $\frac{1}{2}$ (C) 1 (D) $\sqrt{2}$ (E) NOTA

27. Solve for x : $\log_3(\log_8(\log_6 x)) = -1$

- (A) 36 (B) 40 (C) 512 (D) 729 (E) NOTA

28. Evaluate: $\begin{bmatrix} 1 & 0 & -2 \\ 4 & -1 & 3 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 0 & 1 \\ 5 & 2 \end{bmatrix}$

- (A) $\begin{bmatrix} -8 & 1 \\ 23 & 14 \end{bmatrix}$ (B) $\begin{bmatrix} -8 & -7 \\ -18 & 14 \end{bmatrix}$ (C) $\begin{bmatrix} -8 & -7 \\ 23 & -7 \end{bmatrix}$ (D) $\begin{bmatrix} -8 & 1 \\ -18 & -7 \end{bmatrix}$ (E) NOTA

29. Evaluate: $\begin{vmatrix} 4 & 7 \\ -2 & 3 \end{vmatrix}$

- (A) 36 (B) 26 (C) 2 (D) -2 (E) NOTA

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30. If $A = \begin{bmatrix} 3 & 0 \\ 5 & -2 \end{bmatrix}$ and $AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, which of the following is a possible value of B ?

I. $\begin{bmatrix} \frac{1}{3} & 0 \\ \frac{5}{6} & -\frac{1}{2} \end{bmatrix}$

II. $\begin{bmatrix} -2 & 0 \\ -5 & 3 \end{bmatrix}$

III. $\begin{bmatrix} -2 & -5 \\ 0 & 3 \end{bmatrix}$

- (A) I only (B) II only (C) III only (D) I & II only (E) NOTA

31. What values of x satisfy the inequality $\left| \frac{4}{1-x} - \frac{2x+1}{3x} \right| > -6$?

(A) $x > -\frac{1}{2}$ or $x < -5$

(B) $-5 < x < -\frac{1}{2}$

(C) $x < -\frac{1}{2}$

(D) $x > -5$

(E) NOTA

32. If M is the least common multiple of the ten smallest positive multiples of three, how many positive integers are factors of M ?

(A) 810

(B) 756

(C) 720

(D) 702

(E) NOTA

33. The sum of the N smallest positive integers is a multiple of 88. What is the smallest possible value of N ?

(A) 10

(B) 11

(C) 22

(D) 32

(E) NOTA

34. What is the sum of the four smallest natural numbers that are each congruent to 4 mod 7?

(A) 52

(B) 58

(C) 60

(D) 65

(E) NOTA

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35. Two cards are drawn from a standard 52-card deck. What is the probability at least one is an odd-numbered card? (note: Aces count as odd-numbered cards, but Jacks, Queens, and Kings do not)

- (A) $\frac{8}{13}$ (B) $\frac{10}{17}$ (C) $\frac{138}{221}$ (D) $\frac{415}{663}$ (E) NOTA

36. When flipping a fair coin, the first three flips are all heads. What is the probability that the next two flips will be heads?

- (A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{3}{4}$ (D) 1 (E) NOTA

37. A bag contains 7 white and 4 black marbles. Two marbles are drawn at random. What is the probability they are of the same color?

- (A) $\frac{26}{55}$ (B) $\frac{27}{55}$ (C) $\frac{28}{55}$ (D) $\frac{29}{55}$ (E) NOTA

38. Which of the following functions might have generated the sequence: 4, 7, 12, 19, 28, ...

I. $f(n) = An + B, \quad A \neq 0$

II. $f(n) = An^2 + Bn + C, \quad A \neq 0$

III. $f(n) = \frac{An^2 + Bn + C}{Dn + E}, \quad A \neq 0, D \neq 0$

- (A) I only (B) II only (C) III only (D) II & III only (E) NOTA

39. Given that $k(m) = 5 - 2m$ and $r(m) = 2m - 5$, what is $k(r(m))$?

- (A) m (B) $15 - 4m$ (C) $10 + 4m$ (D) $25 - 4m^2$ (E) NOTA

40. What is the fifth term of the arithmetic sequence whose first three terms are $x - 3$, $2x + 1$, and $x + 18$?

- (A) 44 (B) $\frac{89}{2}$ (C) 45 (D) $\frac{91}{2}$ (E) NOTA