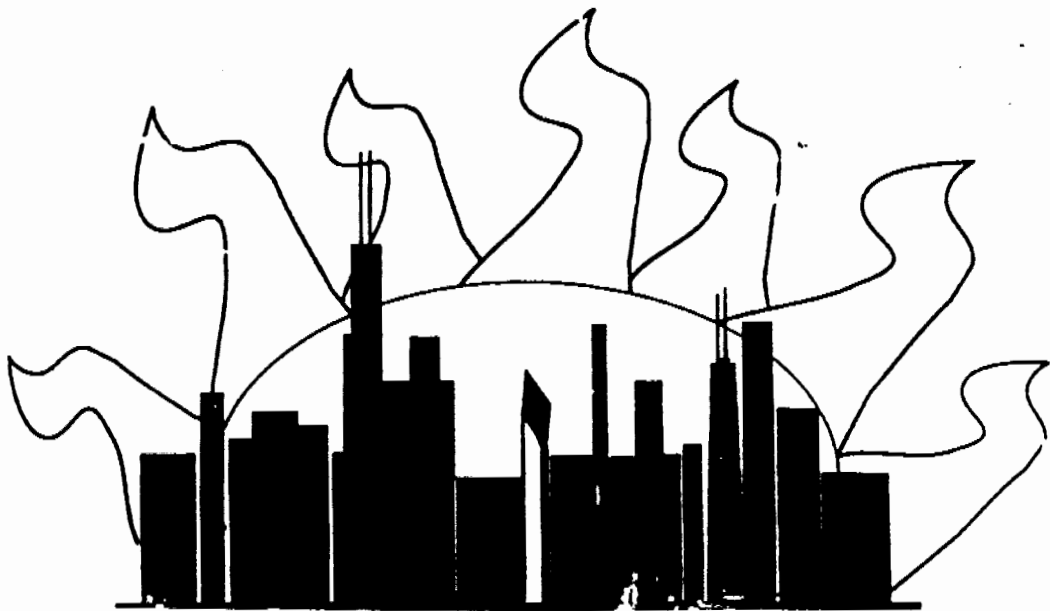


Relay Tests



Mu Alpha Theta National Convention
Chicago 1998

Round 1

1. If $x + 3$, $2x - 7$, and $5x + 3$ are the first three terms of an arithmetic progression, find x .
2. Find the smallest positive value of k so that $x^2 + kx + \text{ANS} + 14 = 0$ has real solutions.

3. Find t , where $\begin{vmatrix} 0 & t & -1 \\ 2 & 4 & -3 \\ 0 & 1 & 5 \end{vmatrix} = \text{ANS}$. Give your answer as a reduced fraction.

4. Let $\sin \theta = |\text{ANS}|$, $\cos \beta = \frac{5}{13}$, $0 \leq \beta \leq \frac{\pi}{2}$, $\frac{\pi}{2} \leq \theta \leq \pi$. Find $\sin(\theta + \beta)$. Give an exact answer.

ANSWERS

1. -10
2. 4
3. $\frac{-3}{5}$
4. $\frac{-33}{65}$

Round 2

1. The points $(5, 7)$, $(3, a + 2)$, and $(7, 11)$ are collinear. Find a .
2. Let $ANS = 2x + 11$. Find the sum of the roots of $5x^2 + 12xy + 3y^2 = 0$.
3. Let α be the angle between the positive x -axis and the line segment with endpoints $(0, 0)$ and $(ANS, 10)$. Find $\sin 2\alpha$. Give your answer as a reduced fraction.
4. The probability that a single dart hits a target is $\frac{ANS}{2}$. Three darts are thrown at the target. What is the probability that exactly one hits the target? Give your answer as a reduced fraction. Assume the dart tosses are independent events with the same probabilities.

ANSWERS

1. 1
2. 20
3. $\frac{4}{5}$
4. $\frac{54}{125}$

Round 3

1. Find the product of the positive roots of $x^3 - 4x^2 - 9x + 36 = 0$.
2. Find x if $\left(\frac{\text{ANS}}{3} + i\right)^2 - (3 + i)^2 = 2i + x$ where $i = \sqrt{-1}$.
3. An isosceles triangle has a perimeter of 17 and a base of length ANS. Find the vertex angle in degrees to the nearest degree.
4. The sum of the three terms of an arithmetic sequence is 4 more than ANS. Find the middle term.

ANSWERS

1. 12
2. 7
3. 89
4. 31

Round 4

1. Find the sum of the non-real roots of $x^3 - 1$.
2. The second term of an arithmetic sequence is ANS and the common difference is 4. What is the 14th term?
3. Find the height of the flagpole which casts a shadow of 2(ANS) meters when the angle of elevation of the sun is 21°. Give your answer, in meters, correct to the nearest tenth of a meter.
4. Let f be a linear function with $f(2) = 7$ and $f(-2) = \frac{[[ANS]]}{12}$. Find $f(3)$.

Note: $[[x]]$ denotes the greatest integer function.

ANSWERS

1. -1
2. 47
3. 36.1
4. 8

Round 1

1. A rectangular box has volume 2000. Its length is twice its width and its height is four times its length. Find the sum of the three dimensions.
2. Batperson goes for a drive in his Batmobile at ANS miles per hour. Two hours later Superperson follows flying at 77 miles per hour. How many hours will it take Superperson to catch up with Batperson?
3. You have two concentric circles with radii ANS and $ANS + 6$. Find the area inside the larger circle and outside the smaller circle.
4. A right circular cone with height 2 has volume ANS . What is its radius?

ANSWERS

1. 55
2. 5
3. 96π
4. 12

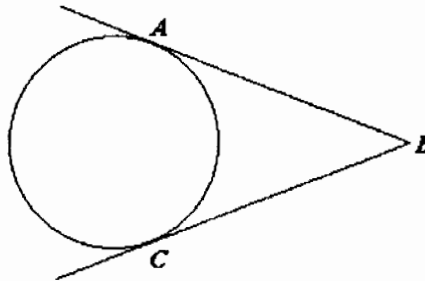
Round 2

1. Solve the following equation:

$$\frac{3}{4}(x-3) - \frac{5}{2}(2x+5) = \frac{7}{8}(3x-1) + 3.$$

Your answer should be a simplified fraction.

2. Ralph is throwing a ball at the broad side of a barn. The random tosses are independent events. The probability that a single toss hits the barn is $|1/\text{ANS}|$. What is the probability that Ralph hits the broad side of the barn at least once in six tosses? Give your answer correct to three significant digits.
3. The sides of a triangle are the three digits of $(\text{ANS} + .002)$. Find the area of this triangle. Give your answer correct to the nearest integer.
4. In the following diagram, BA and BC are tangent to the circle. Find, in **radians**, the measure of the smaller arc from A to C , if the degree measure of $\angle ABC$ is $(\text{ANS})^\circ$. Give your answer correct to four significant digits.



ANSWERS

1. $-\frac{27}{11}$
2. .957
3. 22
4. 2.758

Round 3

1. Find the sum of the roots of $(x - 1)(x + 2) = (2x - 3)(x + 4)$.
2. Find x where $\log_2(x - \text{ANS}) = 3$.
3. A jar has 5 red marbles and 6 green marbles. ANS marbles are picked at random. What is the probability that we get the same number of red marbles as we get green marbles? Give your answer as a reduced fraction.
4. If $\tan \Theta = \text{ANS}$, find $\cos(2\Theta)$. Give an exact answer.

ANSWERS

1. -4
2. 4
3. 5/11
4. 48/73

Round 4

1. The points $(5, 7)$, $(3, \alpha + 2)$, and $(7, 11)$ are collinear. Find α .
2. Let $\text{ANS} = 2x + 11$. Find the sum of the roots of $5x^2 + 12xy + 3y^2 = 0$.
3. Let α be the angle between the positive x -axis and the line segment with endpoints $(0, 0)$ and $(\text{ANS}, 10)$. Find $\sin 2\alpha$. Give your answer as a reduced fraction.
4. The probability that a single dart hits a target is $\frac{\text{ANS}}{2}$. Three darts are thrown at the target. What is the probability that exactly one hits the target? Give your answer as a reduced fraction. Assume the dart tosses are independent events with the same probabilities.

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Round 2

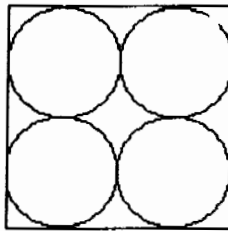
1. Find the mean of the mean and mode of the following data:
88, 24, 32, 21, 84, 26, 21, 23, 49, 22
2. How much longer, in hours, would it take a boat to travel 156 miles upstream as opposed to 162 miles down stream if the river is moving at 6 mph and the boat has a speed of ANS mph in still water?
3. Suppose ANS different circles in three dimensions have the same center and radius. What is the minimum number of points at which they intersect?
4. ANS circles of the same radius are drawn on a 10 x 12 piece of paper, all tangent to the same edge of the paper. If two different circles do not intersect in two or more different points, what is the least amount of area not covered by the circles?

ANSWERS

1. 30
2. 2
3. 2
4. $120 - 18\pi$

Round 3

1. A regular six-sided die is one with the numerals 1, 2, 3, 4, 5, 6 on its sides. If two regular fair dice are tossed what is the probability that the sum of the top faces will be at least 10? Give your answer as a reduced fraction.
2. Simple interest, I , on a loan is given by the formula $I = PRT$ where P is the amount loaned in dollars, R is the interest rate per year, and T is the number of years. Find the simple interest in dollars, on a loan of \$12,000 for ANS years at a rate of 5% per year.
3. Four circles with the same diameter just fit inside a square as shown. Find the area of the region which is inside the square but outside the circles given the area of the square is ANS.



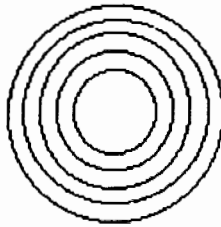
4. A yardstick and a tree cast shadows on level ground. How many feet high is the tree if the shadow of the yardstick is 10 feet and the shadow of the tree is ANS + 25π feet?

ANSWERS

1. $\frac{1}{6}$
2. 100
3. $100 - 25\pi$
4. 30

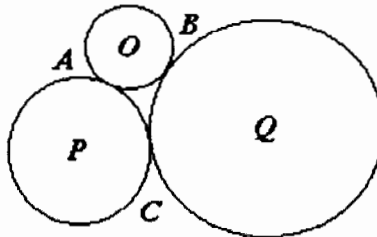
Round 4

1. Three squares are nested, one inside the other. Each of the two smaller squares have the property that their vertices are the midpoints of the edges of the next larger squares. If the area of the smallest square is 1, what is the area of the largest square?
2. Find the units digit of $(ANS)^{(1998 + 1)}$.
3. The following diagram shows several concentric circles. The difference between the radii of two adjacent circles is 3 feet and the radius of the inner circle is 3 feet. A person walks around the inner circle at a rate of ANS feet per second. At what rate, in feet per second, must another person run around the outside track so that the two people complete one revolution in the same amount of time?



4. Suppose O , P , and Q are centers of circles with radii of 2, $\frac{ANS}{5}$, 6, respectively.

These circles are mutually tangent at the points A , B , C , as shown. Determine the length of the segment AB .



ANSWERS

1. 4
2. 4
3. 20
4. $2\sqrt{2}$