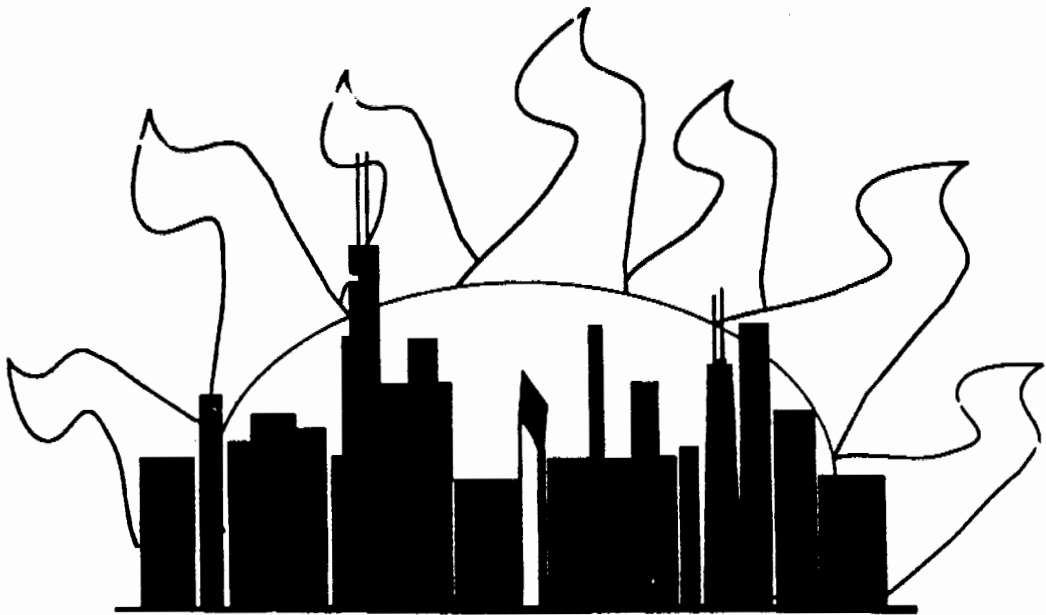


Alpha Division

Topic Test 1

Vectors



**Mu Alpha Theta National Convention
Chicago 1998**

General Instructions:

1. Unless otherwise stated all answers should be written as decimals.
2. If you are asked to give your answer as a fraction, please give your answer in $\frac{a}{b}$ form where a and b are relatively prime.
3. Boldface notation indicates vectors.

Questions

1. What is the product of the values of a such that $\mathbf{A} = a\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\mathbf{B} = 2a\mathbf{i} + a\mathbf{j} - 4\mathbf{k}$ are perpendicular?
2. Find the acute angle, in degrees, between the vectors $\mathbf{A} = 2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{B} = 6\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$. Give your answer to the nearest degree.
3. Simplify $2\mathbf{A} + \mathbf{B} + 3\mathbf{C} - \{\mathbf{A} - 2\mathbf{B} - 2(2\mathbf{A} - 3\mathbf{B} - \mathbf{C})\}$. Write your answer as the sum of the coefficients of \mathbf{A} , \mathbf{B} and \mathbf{C} .
4. Evaluate $\mathbf{k} \cdot (\mathbf{i} + \mathbf{j})$.
5. Two vectors of magnitude 6 are orthogonal. What is the magnitude of the resultant sum of these two vectors?
6. Let $\mathbf{v}_1 = \langle 3, 5 \rangle$, $\mathbf{v}_2 = \langle 1, 7 \rangle$ and $\mathbf{v}_3 = \mathbf{v}_2 - \mathbf{v}_1$. Find the acute angle, in degrees, between \mathbf{v}_3 and \mathbf{v}_1 . Note: an alternate notation for \mathbf{v}_1 is $3\mathbf{i} + 5\mathbf{j}$ and \mathbf{v}_2 is $\mathbf{i} + 7\mathbf{j}$. Give answer rounded to four significant digits.
7. Evaluate $(2\mathbf{i} - 3\mathbf{j}) \cdot [(\mathbf{i} + \mathbf{j} - \mathbf{k}) \times (3\mathbf{i} - \mathbf{k})]$.
8. An automobile travels 3 miles due north, then 5 miles northeast. What is the magnitude, in miles, of the resultant vector? Give your answer rounded to four significant digits.
9. Determine the value of a so that $\mathbf{A} = 2\mathbf{i} + a\mathbf{j} + \mathbf{k}$ and $\mathbf{B} = 4\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$ are perpendicular.
10. Find the largest of the three acute angles, to the nearest degree, which the line joining the points $(1, -3, 2)$ and $(3, -5, 1)$ makes with each of the coordinate axes.

11. Find the area of a parallelogram having diagonals defined by $\mathbf{A} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and $\mathbf{B} = \mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$.
12. Two boats are at the same place at 5:00 a.m. on a certain day. Starting at 5:00 a.m., the first boat is sailed 7.23° to the west of north at a rate of 11.44 miles per hour. The second boat does not move until 7:00 a.m. on the same day at which time it is sailed at 12.38 miles per hour in a direction 10.25° to the east of north. What is the distance, in miles, between the boats at 11:30 a.m. on the same day?
13. Find the sum of all values of m such that the vector $4\mathbf{i} + m\mathbf{j} + \mathbf{k}$ is perpendicular to the vector $m\mathbf{i} + (2m+1)\mathbf{j} - 3\mathbf{k}$. Give your answer as a simplified fraction in lowest terms.
14. The two planes $2x + 3y - 4z = 12$ and $x - y + z = -3$ intersect in line L . The line whose equation is $\frac{x+2}{-2} = y-1 = \frac{z-4}{7}$ and line L intersect at point (a, b, c) . Find $a + b + c$.
15. Two sides of a triangle are formed by the vectors $\mathbf{A} = 3\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$ and $\mathbf{B} = 4\mathbf{i} - \mathbf{j} + 3\mathbf{k}$. Determine the smallest angle to the nearest degree.
16. A plane contains the points $(1, 2, -3)$, $(0, 4, 6)$ and $(2, 3, -4)$. Find a vector perpendicular to the plane in the form $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ where a, b and c are relatively prime integers with $a > 0$. Give your answer as the value of $a + b + c$.
17. Find the shortest distance from $(6, -4, 4)$ to the line joining $(2, 1, 2)$ and $(3, -1, 4)$.
18. Find the length of the projection of the vector $2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$ on the vector $\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$. Write your answer as a simplified fraction in lowest terms.
19. Two radio stations receive a distress message from an island. Station A is 120 miles due north of Station B. Station A receives the message at a bearing of $S 50^\circ E$ while Station B receives it at a bearing of $N 47^\circ E$. How many minutes will a helicopter flying at 110 mph take to reach the island from Station A? Give your answer rounded to four significant digits. [Note: $S 50^\circ E$ means 50° east of due south.]
20. Find the volume of the parallelepiped whose edges are represented by the vectors:

$$\mathbf{A} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$$

$$\mathbf{B} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$$

$$\mathbf{C} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$$